Chapter 7
Conservation of Energy

Conservative force
Non-conservative force
potential energy & potential function
Mechanical energy conservation

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Conservative and Nonconservative Forces

- A force is conservative if the work done by the force does not depend on the path taken as the particle moves from one position to another. (only depends on the initial and final points.) In other words,
  - A force is conservative if the work it does on a particle is zero when the particle moves around any closed path, returning to its initial position.

Example of a nonconservative force: friction

- Friction always retards the motion, so the work it does around a closed path is nonzero.
  so the longer the path is, the more work done.
Examples of conservative forces:

- Spring force
- Gravity

Conservative Forces:

- In general, if the work done does not depend on the path taken, the force involved is said to be conservative.

Performing integration:

- Gravity is a conservative force: \( W_g = Gm \left( \frac{1}{R_2} - \frac{1}{R_1} \right) \)
- Gravity near the Earth’s surface: \( W_g = -mg\Delta y \)
- A spring produces a conservative force: \( W_s = -\frac{1}{2}k(x_2^2 - x_1^2) \)

Potential Energy

- For any conservative force \( F \) we can define a potential energy function \( U \) in the following way:

\[
W = \int F \cdot d\vec{r} = -\Delta U
\]

The work done by a conservative force is equal and opposite to the change in the potential energy function.

This can be written as:

\[
\Delta U = U_2 - U_1 = -\int F \cdot d\vec{r}
\]

Potential Energy & Equilibrium:

For a given potential function \( U(x) \), one can find the force:

- Its derivative w.r.t. \( x \) is \( F_x \). At equilibrium: \( dU(x)/dx = 0 \).

- Its second derivative tells its stability \( dF(x)/dx > 0 \) or \( < 0 \)

- \( F \) physical: determines the motion \( F = ma \).

- \( U \) auxiliary: up to an arbitrary const.
Conservative Forces and Potential Energies

<table>
<thead>
<tr>
<th>Force</th>
<th>Work</th>
<th>Change in P.E.</th>
<th>P.E. function U</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_g = -mg \hat{y}$</td>
<td>$-mg(y_2-y_1)$</td>
<td>$mg(y_2-y_1)$</td>
<td>$mg\hat{y} + C$</td>
</tr>
<tr>
<td>$F_G = -\frac{GMm_2}{R}$</td>
<td>$GMm_2 \left(\frac{1}{R_2} - \frac{1}{R_1}\right)$</td>
<td>$-GMm_2 \left(\frac{1}{R_2} - \frac{1}{R_1}\right)$</td>
<td>$\frac{GMm_2}{R} + C$</td>
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<tr>
<td>$F_s = -kx\hat{x}$</td>
<td>$\frac{1}{2}k(x_2^2 - x_1^2)$</td>
<td>$\frac{1}{2}k(x_2^2 - x_1^2)$</td>
<td>$\frac{1}{2}kx^2 + C$</td>
</tr>
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(R is the center-to-center distance, x is the spring stretch)

**Question**

You lift a 10-kg bag of flour from the floor to a shelf 2m above the floor. Which of the following statements are true?

A. You increased the gravitational potential energy and performed negative work on the bag.
B. You increased the gravitational potential energy and the earth performed positive work on the bag.
C. You increased the gravitational potential energy and the earth performed negative work on the bag.
D. You decreased the gravitational potential energy and the earth performed positive work on the bag.

You did positive work on the bag (displacement along $F$). Gravity did negative work on the bag (opposite to displacement). The gravitational potential energy of the bag increased.

**A woman runs up a flight of stairs.** The gain in her gravitational potential energy is $U$. If she runs up the same stairs with twice the speed, what is her gain in potential energy?

- A) $U$
- B) $2U$
- C) $U/2$
- D) $4U$
- E) $U/4$

Then what is the difference?

$U = W = \text{f d}$, But $P = W/t = \text{f v}$.

**Mechanical Energy Conservation:**

$E_{\text{mech}} = K_{\text{sys}} + U_{\text{sys}}$

**Definition — Total Mechanical Energy**

$W_{\text{out}} = \Delta E_{\text{mech}} = W_{\text{nc}}$

**Work-Energy Theorem for Systems**

In the absence of Non-Conservative Forces:

$E_{\text{mech, i}} = E_{\text{mech, f}}$ (or $K_i + U_i = K_f + U_f$)
Question: Falling Objects

Three objects of mass \( m \) begin at height \( h \) with velocity zero. One falls straight down, one slides down a frictionless inclined plane, and one swings at the end of a pendulum. What is the relationship between their speeds when they have fallen to height zero?

The only work is done by gravity, which is a conservative force: \( v_f = v_i = \sqrt{2gh} \)

<table>
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<tr>
<th>Free Fall</th>
<th>Frictionless incline</th>
<th>Pendulum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_f &gt; v_i &gt; v_p )</td>
<td>( v_f = v_i = v_p )</td>
<td></td>
</tr>
</tbody>
</table>

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A ball, initially at rest, is dropped from a place 1.5 m above the floor and is observed to have a velocity \( V \) just before it hits the floor. If instead, the ball is dropped from a place that is 0.5 m above the floor, the velocity just before it hits the floor is:

- A) 33% of \( V \)
- B) 50% of \( V \)
- C) 58% of \( V \)
- D) 65% of \( V \)
- E) 71% of \( V \)

\[ \frac{mg \cdot 0.5}{\frac{1}{2}mv^2} = \frac{0.5}{1.5} = 57.7\% \]

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Loop-the-loop

How high up from the ground must the cart stop to complete the loop-the-loop?

To stay on the track, cart must have large enough speed at the top of the circle so that its centripetal acceleration is at least \( g \):

\[ \frac{v^2}{R} \geq g \]

So, at the top of the loop, \( E_p = \frac{1}{2}mv^2 \geq \frac{1}{2}mgR \).

Therefore, the potential energy at the very start must be bigger than \( E_p \) (top) = \( E_p \) (2R) + \( E_p \) = \( mg \) (2R) + \( \frac{1}{2} \) \( mg \) R = 5\( mg \) R/2.

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• A 1 kg block slides 4 m down a frictionless plane inclined at 30 degrees to the horizontal. After reaching the bottom, it slides along a frictionless horizontal plane and strikes a spring with spring constant \( k = 314 \) N/m. How far is the spring compressed when it stops the block?

The decrease in the block’s height is \( h = L \sin \theta \), so the magnitude of the change in potential energy when the block slides down is \( E_p = E_p = mg \ L \sin \theta \).

The spring compresses until its potential energy has that value, so \( \frac{1}{2}kx^2 = mg \ L \sin \theta \) \( \Rightarrow \) \( x = (2mgL \sin \theta / k)^{1/2} = 35 \) cm.

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Summary: Non-conservative Forces and Energy Conservation

- When non-conservative forces are present, the total mechanical energy of the system is not constant.
- The work done by all non-conservative forces acting on parts of a system equals the change in the mechanical energy of the system:
  \[ W_{nc} = \Delta \text{Energy} \]
  If all forces are conservative, \( W_{nc} = 0 = \Delta \text{Energy} \)
- In equation form:
  \[ \Delta \text{Energy} = W_{nc} = (KE_f - KE_i) + (PE_f - PE_i) \]
  \[ = (KE_i + PE_i) - (PE_f + KE_f) \]
- The energy is not really lost but is generally transformed into a form of non-mechanical energy such as thermal energy.