1) Quickies:
   a. Calculate $\langle jm + 2 | J^z | jm \rangle$ for $j = 3/2, m = -1/2$.
   b. Find $[p, x^2]$
   c. In a basis $|A\rangle, |B\rangle, |C\rangle$, an operator $W = a|A\rangle\langle A| + \beta|B\rangle\langle B| + \gamma|B\rangle\langle C| + \gamma|C\rangle\langle B|$. Write the matrix form of $W$ in this basis.

2) Neutrinos, when they are created in weak interactions, are produced in so-called "flavor" eigenstates, i.e. $|e\rangle$ or $|\mu\rangle$ for electron-type or muon-type neutrinos. However, their energy eigenstates are $|1\rangle = \cos \theta |e\rangle + \sin \theta |\mu\rangle$, and $|2\rangle = ?$ with energies $E_1, E_2$.
   a. Use orthogonality to deduce the amplitudes $\langle e|2\rangle$ and $\langle \mu|2\rangle$.
   b. At $t = 0$ a muon-neutrino is produced. Find the probability of observing an electron neutrino as a function of time.

3) A 131-Xe nucleus in a magnetic field has 4 states with energies $E_m = m\mu B$, $m = \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}$. In the presence of an electric field gradient, it also experiences a quadrupole interaction

   \[
   V = \frac{q}{8} \begin{pmatrix}
   1 & -2\sqrt{3} & \sqrt{3} & 0 \\
   -2\sqrt{3} & -1 & 0 & \sqrt{3} \\
   \sqrt{3} & 0 & -1 & 2\sqrt{3} \\
   0 & \sqrt{3} & 2\sqrt{3} & 1
   \end{pmatrix}.
   \]

   Find the energies of the states to first order in $Q$.

4) A particle of mass $m$ moves in a potential $V(x)$. Suppose we define an operator $A = ip/\sqrt{2m} + W(x)$, where $p$ is the momentum operator and $W(x)$ is a function of $x$ to be determined. Show that the Hamiltonian can be written in the form $H = A^\dagger A$, and find a 1st order differential equation relating $W(x)$ and $V(x)$.
5) (Take home) Consider a harmonic oscillator of resonance frequency $\omega$, with an added perturbation $V = \alpha \hbar \omega x^4$, where $\alpha$ is a dimensionless number and $x$ is defined in BD 7.24. We wish to know the energy shift of state $\ket{0}$ due to $V$.

a. Use raising and lowering operators to calculate the matrix elements $\langle 0|V|0\rangle$, $\langle 2|V|0\rangle$, $\langle 4|V|0\rangle$. Use perturbation theory to calculate the energy shift to 2nd order in $V$.

b. An alternate approach to the problem is to set up a matrix representation of $V$ using the basis $\ket{0} \ldots \ket{4}$. An easy way to do this in Mathematica is to write $a$ and $a^\dagger$ in matrix form, then generate $V$ (and then $H$) by addition and matrix multiplication. Do this.

c. You should have found that by reordering the basis states, $H$ can be rewritten in the form $H = \begin{pmatrix} A & 0 \\ 0 & S \end{pmatrix}$ where $A$ is a 2X2 matrix and $S$ is a 3X3 matrix. There is a good physics reason for this; what is it?

d. Plot the lowest eigenvalue of the Hamiltonian for $0 < \alpha < 0.2$ and compare to your perturbation theory result.