Phys 448 Exam 1

1) A particle of mass $m$ moves in the potential shown below.

\[ V(\infty) = V_0 \]

a. Make careful sketches of the wavefunctions of the lowest state and the first excited state. Show as many details as you can without knowing the energies of the states.

b. Defining $q^2 = \frac{2ma^2V_0^2}{\hbar^2}$, how many bound states will there be in this potential? (This can be answered exactly without explicitly solving the problem—think carefully about the shape of the highest energy bound wavefunction.)

c. Find the quantization condition and explain how you might go about solving it.

2) A beam of particles of mass $m$ and energy $E > 0$ encounters a potential $V(x) = \frac{\hbar^2 q}{2m} \delta(x)$. Find the transmission probability.

3) Evaluate the uncertainty product for a mass $m$ particle with the wavefunction $e^{-x|x|}$, bound in a $\delta(x)$ potential. Useful facts:

\[ \int_0^\infty x^n e^{-x} = n!, \quad \langle V(x) \rangle = -\frac{\hbar^2 k^2}{m} \]

4) Use any method you wish to estimate the energy of the lowest energy level of a Rb atom (mass 87 amu) moving in the potential

\[ V(x) = V_0 \left( \frac{x}{a} \right)^4 \]

with $V_0 = 1$ mK, $a = 1$ µm. 1 amu = 931.6 MeV.