1) T 6.2
2) T 6.4
3) T 6.5

4) Consider a beam of particles with known kinetic energy \( E \). In three dimensions, the Hamiltonian becomes 
\[
H = \frac{1}{2m} \left( \hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2 \right) = -\frac{\hbar^2}{2m} \nabla^2
\]
with eigenfunctions \( e^{i\mathbf{p} \cdot \mathbf{r}} \), \( \mathbf{r} = (x, y, z) \), \( \mathbf{p} = (p_x, p_y, p_z) \). The beam is travelling in the z-direction. Assume that the spatial wavefunction can be written \( \psi(\mathbf{r}) = f(x, y, z) e^{ikz} \), where \( \frac{\partial f}{\partial z} \ll k f \). Plug this assumption for \( \psi(\mathbf{r}) \) into the Schrödinger equation, and use the deBroglie relation between \( k \) and \( E \) to find a new “paraxial” wave equation for \( f \). It should involve a single derivative with respect to \( z \), and second partial derivatives with respect to \( x \) and \( y \).

5) Use Mathematica to verify that the function \( f(x,y,z) = \frac{1}{q(z)} e^{i \frac{k (x^2 + y^2)}{2q(z)}} \) solves this equation. Find \( q(z) \) using the boundary condition \( f(x, y, 0) = e^{-(x^2+y^2)/w_0^2} \), where \( w_0 \) is a constant.

6) Defining a width parameter \( w(z) \) via \( |f(x, y, z)|^2 = C(z) e^{-(x^2+y^2)/w^2} \), find and plot \( w(z) \). At what distance scale does the beam begin to spread?