6.1. \( \mathbf{H} = \begin{pmatrix} E_0 & -a & 0 \\ -a & E_0 & -a \\ 0 & -a & E_0 \end{pmatrix} \)

\[ \mathbf{M} : \quad E = E_0 + \varepsilon - \sqrt{a}, \varepsilon, \sqrt{a} \]

\[ |0\rangle = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix}, \quad |\uparrow\rangle = \frac{1}{2} \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}, \quad |\downarrow\rangle = \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \]

b) ground state \( L: \frac{1}{4}, C: \frac{1}{2}, R: \frac{1}{4} \)

c) \( \langle 0 | L \rangle = -\frac{\sqrt{2}}{4}, \quad \langle -\downarrow | L \rangle = \frac{1}{2}, \quad \langle +\downarrow | L \rangle = \frac{1}{2} \)

\[ P(E_0) = \frac{1}{4}, \quad P(E_0 + \sqrt{2a}) = P(E_0 - \sqrt{2a}) = \frac{1}{4} \]

\[ \langle E \rangle = E_0 + \frac{1}{2} \cdot 0 + \frac{1}{4} (\sqrt{2a}) + \frac{1}{4} (-\sqrt{2a}) = E_0 \]

\[ \langle E^2 \rangle = \frac{1}{2} E_0^2 + \frac{1}{4} (E_0 + \sqrt{2a})^2 + \frac{1}{4} (E_0 - \sqrt{2a})^2 \]

\[ \Delta E = a \]

6.2. \( \mathbf{H} = \begin{pmatrix} 0 & -A & -A \\ -A & 0 & A \\ -A & -A & 0 \end{pmatrix} \)

Ammonia \( \begin{pmatrix} 0 & -A \\ -A & 0 \end{pmatrix} \) \( \Rightarrow \quad E = \pm A \)

quite similar
$$M: \langle \phi_1 | H | \phi_1 \rangle = -2a - a^2$$

$$\Rightarrow \Delta E = \sqrt{4a^2 + a^2} = 0$$

$$\langle \phi_2 | H | \phi_2 \rangle = a$$

$$\Rightarrow \Delta E = 0$$

$$\langle \phi_2 | H | \phi_2 \rangle = a^2$$

$$E = \{ \pm a, a^2 \}, \quad |E \rangle = \{1, 1, 1\}$$

$$11' = (1, 0, -1)$$

$$11' = (-1, 1, 0)$$

Since $|11' \rangle$ both have $E = a$, any linear combination of them is also an eigensate. The basis is not unique.

d) energy splitting is $3a = 2.25 \text{ eV}$, so yellow light is absorbed. So transmitted light looks violet.

e) New $H = \begin{pmatrix} A & A & A \\ A & 0 & A \\ A & A & 0 \end{pmatrix}$

$$M: \text{eigenvalues } a, a - \frac{a}{2} \pm \frac{1}{2} \sqrt{9a^2 + 2da + A^2}$$

$$\Delta \gg a, \quad \text{get } \quad -\frac{a}{2} \pm \frac{3a}{2} = a, -2a \text{ as before}$$

$$\Delta \gg 2a, \quad \text{get } \quad 0, 0, \Delta$$

f) $\nu_1 = a - (\frac{A}{2} - \frac{a}{2} \pm \frac{1}{2} \sqrt{9a^2 + 2da + A^2}) = \frac{3a}{2} - \frac{a + 1}{2} \sqrt{9a^2 + 2da + A^2}$

$$\nu_2 = \sqrt{9a^2 + 2da + A^2}.$$
\[ h\nu_1 = \frac{3a - \Delta}{2} + \frac{h\nu_2}{2} \]

\[ 3a - \Delta = 2h\nu_1 - h\nu_2 = 2(2.75\text{eV}) - 2.75\text{eV} = 1.25\text{eV} \]

\[ \Delta = 2.75\text{eV} - 1.25\text{eV} = 1.0\text{eV} \]

Check: \[ \sqrt{q^2 + 2\Delta q + \Delta^2} = 2.75\text{eV} \]

with \( \Delta = 1.0\text{eV} \), works well.

3) \[ U = e^{-i\frac{A t}{h}} |10\rangle |01\rangle e^{i\frac{A t}{h}} |10\rangle |01\rangle \]

\[ |10\rangle = |1\rangle |e\rangle - |1\rangle |e\rangle \]

\[ |e\rangle = \frac{|1\rangle + |e\rangle}{\sqrt{2}} \]

in R, L basis, e.g. \[ \langle R1|0\rangle = -\frac{1}{2} e^{-i\frac{A t}{h}} + \frac{1}{2} e^{i\frac{A t}{h}} \]

\[ U = \begin{pmatrix} \cos\frac{A t}{h} & -i \sin\frac{A t}{h} \\ i \sin\frac{A t}{h} & \cos\frac{A t}{h} \end{pmatrix} = i \sin\frac{A t}{h} \]

\[ \pi\text{-pulse} (01) \cdot U \cdot (01) = \cos\frac{A t}{h} + i \sin\frac{A t}{h} = i \]

\[ P_L = \cos^2 \frac{A t}{h} = \frac{1}{2} - \frac{1}{2} \sin^2 \frac{2A t}{h} \Rightarrow \frac{1}{2} - \frac{1}{2} \sin 2\pi ft \]

\[ \Rightarrow f_r = \frac{2A}{h} \]

\[ \pi\text{-pulse} \quad \psi = \begin{pmatrix} 0 \\ i \end{pmatrix} \]

\[ 2\pi\text{-pulse} \quad \psi = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \]
4) \( \pi / 2 \)  \[ u = \begin{pmatrix} \sqrt{2} & i \sqrt{2} \\ i \sqrt{2} & \sqrt{2} \end{pmatrix} \]

\[
\begin{align*}
(0, i \cdot U \cdot \begin{pmatrix} e^{-i \pi / 2} & 0 \\ 0 & e^{i \pi / 2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (i \sqrt{2} \quad \sqrt{2}) \cdot \begin{pmatrix} e^{-i \pi / 2} \\ 0 \end{pmatrix} \cdot (0 \\ e^{i \pi / 2}) \\
= \frac{i e^{-i \pi / 2}}{2} + \frac{i e^{i \pi / 2}}{2} = i \cos \frac{\pi T}{2h}
\end{align*}
\]

\[
\therefore \quad \rho_L = \sum \cos^2 \frac{\pi T}{2h} = \frac{1}{2} + \frac{1}{2} \cos \frac{\pi T}{h}
\]