Phys 448 Exam 2.

Work 3 problems, including 4), in class. Work the other two problems at home, due Thurs. at 5 pm. Same rules as Exam 1.

1) Let \( W = \alpha |A\rangle\langle A| + \gamma |B\rangle\langle C| + \gamma |C\rangle\langle B| \), where
\[ |A\rangle = |1\rangle, \quad |B\rangle = \frac{|2\rangle + i|3\rangle}{\sqrt{2}}, \quad |C\rangle = \frac{|3\rangle + i|2\rangle}{\sqrt{2}}. \]
   a. What are the conditions on \( \alpha \) and \( \gamma \) so that \( W \) is Hermitian?
   b. Write the matrix form of \( W \) in the \( |1\rangle, |2\rangle, |3\rangle \) basis.

2) Three operators \( A_j \) obey the commutation relations
\[ [A_1, A_2] = iA_3, \quad [A_2, A_3] = iA_1, \quad [A_3, A_1] = iA_2. \]
   a. Calculate \( [A_3, A_x^2 + A_y^2 + A_z^2] \)
   b. The eigenvalues of \( A_3 \) are non-degenerate numbers \( a \) with associated eigenstates \( |a\rangle \). Show, using commutation relations, that \( (A_1 + iA_2)|a\rangle \) is an eigenstate of \( A_3 \) with eigenvalue \( a' = ? \).

3) A 2-level system has a ground state \( |\psi\rangle = \left(\begin{array}{c} 0 \\ 1 \end{array}\right) \). Consider two operators
\[ \sigma_x = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right) \quad \text{and} \quad \sigma_y = \left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array}\right). \]
   a. Calculate \( [\sigma_x, \sigma_y] \).
   b. Check that the product \( \Delta \sigma_x \Delta \sigma_y \) obeys the uncertainty principle.

4) (in class) An atom with energy states \( |e\rangle, |g\rangle \) (energies 0, \( h\omega_0 \)) interacts with resonant light so that
\[ H = \left(\begin{array}{cc} 0 & 2\epsilon \cos \omega t \\ 2\epsilon \cos \omega t & \hbar\omega_0 \end{array}\right) \]
   a. Write the effective Hamiltonian in the rotating wave approximation assuming the Bohr condition is exactly obeyed.
   b. At \( t = 0 \) the atom is in state \( |g\rangle \). Calculate the probability for the atom to remain in state \( |g\rangle \) as a function of time. Show your work.

5) (Take home) You wish to calculate the lowest energy level of a particle moving in the potential \( V = V_0 |x| / b \). You decide to represent the Hamiltonian in a basis of simple harmonic oscillator wavefunctions with
\[ |n\rangle = A_n H_n \left(\frac{z^2}{b^2}\right) e^{-\frac{z^2}{2b^2}}, \quad n = 0, 2, 4, 6, 8 \quad \text{(Why only evens?)} \]
   a. Find the matrix representation of the Hamiltonian in this basis. Hint: use \texttt{NIntegrate} to do the integrals. You can, if you wish, use \texttt{creation/annihilation} operators to calculate the kinetic energy matrix but for the potential energy matrix it is best to do the integrals numerically.
   b. Compare your lowest eigenvalue of your approximate Hamiltonian to the exact result \( E_0 = 0.8086 V_0 \left(\frac{\hbar^2}{mV_0 b^2}\right)^{1/3} \).
   c. Plot your wavefunction and compare to mine, which you can find at
   \( \text{http://www.physics.wisc.edu/~tgwalker/448-9Mathematica} \) under the name \texttt{psignd.nb}.\)