Phys 448 Exam 1

Instructions: Solve Problem #1, and 2 out of #2, #3, & #4 in class. Solve Problem #5, plus the one you didn’t do in class, as take-home. All work must be solved by yourself alone. Any non-human resources you wish may be used for the take-home work. Take-home problems are due in the homework box at 5 p.m. on Thursday afternoon. Do not be late.

Integrals: Let \( I(p) = \int_{-\infty}^{\infty} s^{2p} e^{-s^2} \). \( I(0) = \sqrt{\pi} \), \( I(1) = \frac{\sqrt{\pi}}{2} \), \( I(2) = \frac{3\sqrt{\pi}}{4} \);

\[
\int_{-\infty}^{\infty} e^{-s^2+tsx} = e^{-u^2/4}\sqrt{\pi}
\]

1) (In class only)
A particle of mass \( m \) moves in the potential

\[
V(x) = \begin{cases} 
\infty & x < 0 \\
v(x) & 0 < x < a \\
0 & a < x
\end{cases}
\]

as shown below. We wish to find the bound states in this potential. The solutions to the Schrodinger equation in the potential \( v(x) \) with energy \( E \) are \( s_B(x), c_B(x) \) and obey \( s_B(0) = 0 \), \( c_B'(0) = 0 \). Use the boundary conditions to find a quantization condition of the form \( E = f(E) \). Explain a strategy to find the solutions to this equation.

2) David Wineland won the Nobel Prize last week for experiments he did using ions that are laser cooled down to the lowest quantum energy level in a simple harmonic oscillator potential. The oscillation frequencies are typically 1 MHz. Calculate \( \langle x^2 \rangle \) for a \( \text{Be}^+ \) ion (mass 9 amu) in the ground state of that potential. 1 amu=931.5 MeV/c^2.
3) A beam of particles of mass $m$ and momentum $p_z > \hbar / w_0$, 

$$\psi(x, y, z = 0) = e^{-\frac{(x^2 + y^2)}{w_0^2}}.$$ 

has a wave-function $\psi(x, y, z = 0) = e^{-\frac{(x^2 + y^2)}{w_0^2}}$. Calculate $\langle x^2 \rangle$ at 

$z = 0$, and at a large distance $z > w_0^2 p_z / \hbar$.

4) A particle moves in a potential that has two properly normalized eigenstates $\psi_0$ and $\psi_1$ with energies $E_0$ and $E_1$. Let $\chi = (\psi_0 - i\psi_1) / \sqrt{2}$ be an eigenstate of an operator $\hat{\mathcal{B}}$ with eigenvalue $b$. At 

$t = 0$ a measurement of $\hat{\mathcal{B}}$ is made and the value $b$ is obtained. A 

second measurement of $\hat{\mathcal{B}}$ occurs at time $t$ later. What is the 

probability that $b$ is obtained a second time?

5) (Take home only) A particle moves in a 2-d potential

$$V(x, y) = \begin{cases} \frac{-\pi \hbar^2}{4ma^2} & -a < x < a \\ 0 & |x| > a \end{cases}$$

We wish to find solutions to this that describe the particle 

moving freely along the $y$-direction with momentum $p_y = \hbar k_y$, but 

confined in the $x$-direction. What are the allowed energies? 

What is the maximum value of $p_y$ allowed for the particle to be 

bound?