Physics 249 Lecture 40, Dec 12th 2012
Reading: Chapter 12
HW 11: due Friday Dec 14th

Reminder: Final is Dec 21st 10:05-12:05
Will distribute grade information after last HW is graded including an approximate curve. Probably Dec 17th.

1) The effect of the EM potential on particle momentums

\[ \Delta p = \int F = e \int E, B = e \int \partial A = eA \]

Thus \( p \to p + eA \)

Quantizing \( ih\partial \to ih\partial + eA \)

Then the effect of adding the potential to the wave equation is:

\[ \partial^2 \phi = m^2 \phi \to (\partial - ieA)(\partial - ieA)\phi = \partial^2 \phi - ie(\partial A + A\partial) - e^2 A^2 = m^2 \phi \]

The \( A^2 \) term can be identified as the energy of the potential and the other terms describes how the potential changes momentums.

We can calculate the probability of the potential to cause a specific change in momentum between any two momentum and energy states of a particle as:

\[ P = \phi_f^* V \phi_i = \phi_f^* V \phi_i = -ie \phi_f^* (\partial A + A\partial) \phi_i \]

Effectively \( V \) changes the initial state into a final state with some probability.

However, we still have to figure out what to put in for \( A \) in this equation. Several observations indicate that the photon carries the EM field.

a) The information of the potential is carried at the speed of light.
b) Examining Maxwell’s equation shows that light can be most simply characterized as a traveling potential wave.
c) Proposing that virtual photons exist within the constraints of the Heisenberg uncertainty principle leads to the correct function dependence of the EM potential.

In the equation above we use the traveling wave solution for the photon from Maxwell’s equation in differential potential form for \( A \).

2) How to calculate the effect of the EM interaction.

In cases where the interaction is high energy and we expect only one photon to be exchanged the equation above can be simply used.
To understand the effect of the EM potential: Given the initial wave function consider all of the possible final state wave functions that are allowed by conservation of momentum and energy. For each possible final state substitute the wave function into the above equation and calculate the probability. You can integrate over the possibilities to find a total probability of interaction. Note for high energy interactions there can be substantial probability to have no interaction. For interactions at lower energies you would have to consider multiple exchanges.

In some cases the answer is deterministic. In Rutherford scattering given the impact parameter the scattering angle can be determined uniquely within the limit of diffraction effects.

In cases that are more unique to particles physics such as e+e-→e+e- where annihilation is possible a there is distribution of possible results.

In general the probability for a single interaction is proportional to: $\alpha^2$

$$\alpha = \frac{k_e e^2}{\hbar c} \approx \frac{1}{137}$$

More accurately the probability that comes from the above calculation is:

$$P \propto \frac{\alpha^2}{(q^2)^2}$$

3) The weak force.

The electromagnetic force only occurs with particles that carry electric change. The force is mediated by the photon, which is electrically neutral. Electromagnetic interactions will conserve electric charge (and the charges of the other forces).

**Weak**

Similarly the weak force only occurs with particles that carry weak flavor charge. Leptons have lepton flavor charge and quarks have quark flavor charge. The weak force is mediated by the W+, W- and Z bosons. The weak force will conserve weak flavor charge of several types (and the charges of the other forces).

The weak interaction is different in that can change the nature of the particles involved from charged to neutral because the some of the weak force carriers have electric charge.
Example neutron decay.

\[
\begin{array}{c}
\text{n} \\
\downarrow \\
\left\{ \begin{array}{c}
u \\
d \\
d \\
\end{array} \right. \quad \rightarrow \left\{ \begin{array}{c}u \\
d \\
u \end{array} \right. \\
\end{array}
\]

Example muon decay:

\[
\begin{array}{c}
\mu^- \\
\downarrow \\
W^- \\
\downarrow \\
\left\{ \begin{array}{c}
\nu_e \\
e^- \\
\nu_\mu \end{array} \right. \\
\end{array}
\]

The other fundamental difference is the force carrier. The weak bosons have masses of 81GeV, W+ or W-, and 91GeV, Z.

The probability of interactions if found to be

\[
P \propto \frac{\alpha^2}{(q^2 - M_{W,Z}^2)^2}
\]

The constant that determines the probability is the same. The probabilities are different because of the factor of the mass of the force carrier, which results in substantially lower the probabilities.
The fact that the probabilities are low prevents the type of continuous exchange over time that can result in a bound state.

4) The strong force.

The strong force only occurs for the quarks, which carry color charge. The force is mediated by the gluon. The gluons have color charge. The strong force will again conserve all the charges.

The strong force also has approximately the same basic constant determining the probability.

However, the fact that the force carrier can interact with itself changes the nature of the interaction essentially making strong interaction very high probability and short range.

At high energies you can still set up a situation in which there is only time for one interaction to take place and extrapolating experimental results to the highest energies (in this case energies at the beginning of the Universe) you find approximately the same fundamental constant governs the strong force.