Physics 249 Lecture 34, Nov 28th 2012
Reading: Chapter 11
HW 10: due Friday Dec 7th

1) beta decay

Much of our knowledge of the nucleus comes from the transitions between nuclear states involving radioactivity. One of the best decays for gaining knowledge is beta decays which convert protons to neutrons and vice-versa. This allows direct comparison of proton and neutron energy levels.

Beta decay. The nucleus emits an electron (or positron) and associated neutrino converting a neutron to a to a proton (or vice versa).

\[ \beta^-: n \rightarrow p^+ + e^- + \bar{\nu}_e, \quad N \rightarrow N-1, \ Z \rightarrow Z+1, \ A \rightarrow A \]

\[ \beta^+: p^+ \rightarrow n + e^+ + \nu_e \quad N \rightarrow N+1, \ Z \rightarrow Z-1, \ A \rightarrow A \]

In these interactions atomic number of the nucleus is changed. This interaction occurs for elements that are away from the line of stability for which it is energetically possible and favorable to decay lowering their total potential energy and becoming (more) stable.

Beta decay is known as a weak nuclear decay process. The weak processes are characterized by very low probabilities for interaction and correspondingly long main life times before decays occur. We will explore the reasons for this in the next chapter on particle physics. Calculating weak interaction probabilities is very complex in this case depending on both the details of the weak interaction and the details of the nuclear structure.

Beta- decay

The more common beta decay is beta- decay and the simplest case is neutron decay.

The free neutron is heavier than the proton and electron. 939.57 MeV > 938.28 + 0.511 MeV making this process possible considering conservation of energy (including mass energy).

For the neutron decay analysis of the momentum (magnitude and direction) of the decay products indicated that their must be an unobserved near massless particle involved in the decay.

To understand this consider that the conservation of energy and the resulting energy available for the kinetic energy of the final state atom, electron and neutrino(typically called the Q) and conservation of momentum. First consider the case of beta- decay where Q can be calculated from the difference in mass of the initial and final state atoms.

\[ Q = M_i c^2 - (M_f c^2 - m_e c^2) - m_e c^2 = M_i c^2 - M_f c^2 \]
We neglect the neutrino mass, which is so small that it is too small to measure with current technology.

If only an electron were produced in the decay then in the rest frame of the initial atom the final atom and electron would have equal and opposite momentum to the final state atom by conservation of momentum. This would result in definite values of the momentum and kinetic energy of the electron that can be computed by applying conservation of momentum and energy.

Instead we observe a range of energies between a minimum and maximum value. This indicates a third particle, the neutrino must be present in the decay. The maximum energy corresponds to the situation where the neutrino is at rest and the atom recoils with a low velocity giving it negligible kinetic energy. If the atom is non relativistic you can understand that momentum \( p = mv \) can be conserved with low very \( v \) due the much larger mass of the atom while kinematic energy is very small since it is proportional to \( v^2 \). In this case almost all the available energy goes into the kinetic energy of the electron. The minimum energy occurs when the electron is at rest and the neutrino and final state atom have equal and opposite momentum. In this case the electron has zero kinetic energy.

Also this decay would have violated conservation of angular momentum if the neutrino was not present. Considering neutron beta decay the neutron, electron and proton are all spin \( \frac{1}{2} \). There is no way a spin \( \frac{1}{2} \) object can decay into two spin \( \frac{1}{2} \) objects and still have the same total angular momentum. If there is another spin \( \frac{1}{2} \) object present angular momentum can be conserved.

Beta+ decay:
There are very few common atoms that undergo beta+ decay. Since the proton is less massive than a neutron this typically would result in a larger mass for the final state particles than the initial particles. For this to happen the unstable atom has to have a substantially higher mass due to potential energy. One example is:

\[
^{40}\text{K} \rightarrow ^{40}\text{Ar} + e^+ + \nu_e
\]

Calculating the Q for beta+ decay

\[
Q = M_i c^2 - (M_f c^2 + m_e c^2) - m_e c^2 = M_i c^2 - M_f c^2 - 2m_e c^2
\]

Many less common nuclei occurring further from the line of stability but within the valley of stability occur.

A second method of converting a proton to a neutron is electron capture. This is possible because the electron has non zero probability for its wave function inside the nucleus. The probability of this interaction happening will depend on the probability of the electron being inside the nucleus as well as the probability for weak interactions.
electron capture: \( p^+ + e^- \rightarrow n + \nu_e \)

where the Q value is:
\[
Q = M_i c^2 + m_e c^2 - M_f c^2 - m_e c^2 = M_i c^2 - M_f c^2
\]

Though this is still less common because the neutron is more massive than the proton.

The A=101 isobar illustrates typically decay patterns. Since \( 0.001u = 0.9315\text{MeV} \) the decay of \( Z=45 \) A=101 isotope must happen via electron capture.

Note that sometimes due to the complex structure of the energy levels for protons and neutrons both types of beta decay and electron capture may be allowed. Examples are 40 K and 108 Ag. Also in some cases single beta decay is energetically forbidden but simultaneous double beta decay may be allowed. An example is 82 Se.

2) Proton and neutron emission.

Far from the line of stability you should be able to create nuclei where it is energetically favorable to just eject the proton or neutron completely. This is because as you move away from the line of stability you are raising the potential energy of the nucleons without raising the binding force as you would if you add more nucleons. The lines that define this region in the N vs. Z planes go from the region of stability where you see more common isotopes and decays and extend to lines known a drip-lines. The drip-lines are theoretical since these isotopes do not every occur in nature and can’t be constructed since they reject the extra nucleons instantaneously since they are so energetically unfavorable. On the proton side the drip-line defining this exists close to the “stable” on region on the edge of the valley of stability, since proton repulsion increases potential energy of adding new protons. On the neutron side the line is further from the region of stability since the extra repulsion doesn’t exist. At the edge of the region of stability some isotopes with long enough lifetimes to be observable which decay via proton an neutron emission exist. There are more proton cases since the proton repulsion makes this type of interaction more likely. Since the mechanism of decay in this case is not weak decay probabilities can be larger and lifetimes smaller.
3) Alpha decay

It can also be energetically favorable to eject an alpha particle, which is an entire Helium nuclei. This can occur because Helium is the more tightly bound (lowest potential energy) small nucleus. Therefore, the ejected helium nucleus is low mass for the number of nucleons, and the total binding energy of the system is not reduced that much since the helium nucleus is small. This is most favorable for large nuclei were removing the helium nucleus actually increases the binding energy per nucleon.

However, the probability is still low, even though it is energetically favored, because the combination of the nuclear and EM potentials for a potential barrier which the alpha particle must tunnel through. Interestingly features such as the energy of the alpha particle and the lifetime can be solved by a quantum barrier treatment.
note that fission into larger atoms is just a more complex extension of this idea. Alpha
decay and fission dominate for larger atoms especially above the 126 magic number,
which ending one of the largest shells, and above which adding any protons or neutrons
tends to be unstable.

4) Types of decay that transmute atoms.

The above types of decays all transmute the atoms changing their $Z$ and allowing
unstable nuclei to become (more) stable nuclei or stable pairs of nuclei. The decay
patterns of the radioactive isotopes within the valley of stability have been mapped out as
shown in the figure below.