Physics 249 Lecture 34, Nov 28th 2012  
Reading: Chapter 11  
HW 9: due Friday Nov 30th

1) Nucleon magnetic moments

You would expect the proton to have a magnetic momentum according to a quantized version of the formula

\[ \vec{\mu} = \frac{q}{2m} \vec{L} \]

Similarly to the electron case we can define a nucleon magneton as (2000 times smaller than the Bohr magneton:

\[ \mu_N = \frac{e\hbar}{2m_p} \]

and would expect a maximum magnetic moment measure along an axis for the proton of 0.5\(\mu_N\) and zero for the neutron. However we find that both the neutron and proton have magnetic moments with values.

Proton: 2.79285\(\mu_N\)  
Neutron: −1.91304\(\mu_N\)

This is not consistent with the expectation and is further evidence of their internal structure.

2) Total nuclear angular momentum and spin-orbit coupling.

The spin and orbital angular momentum of each nucleon combine to form a total angular momentum and can cause an energy splitting depending on whether they are aligned or anti-aligned.

As with the electrons the effect there is only a 2 fold splitting

J=L+1/2, J=L-1/2

However, there are two main differences.

a) In the electron spin orbit effect the J=L+1/2 splits upward because the electron sees a magnetic field from the protons that was oriented upward and had a magnetic momentum oriented downward due to the negative charge of the electron giving a positive contribution from \(U = -\vec{\mu} \cdot \vec{B}\). Anti-alignments of elector spin and orbital angular momentum led to a negative contribution to the potential energy.
By analogy to the atomic case the protons are also orbiting relative to the nuclear charge distribution. The dominant portion of the nuclear charge distribution that the proton and neutrons see, that at the shortest distance, is actually their own charges, which is a factor in the non-intuitive splitting for neutrons. The protons have a magnetic moment oriented upward and will see a negative contribution to the potential energy when aligned. In neutrons you might expect the opposite but this turns out not to be the case.

b) The magnetic field of a dipole source ( a circular current) is proportional to $1/r^3$. This mean that the spin orbit effect that the protons and neutrons see will be much larger then the spin orbit effect in the electron case. The splitting will often be larger than the intrinsic $n+l$ energy levels.

On splitting. We can understand how many nucleons can fill each slit energy level by examining the addition of spin and orbital angular momentum.

For the p stand

$L=1$  $S=1/2$

$J= 1/2, 3/2$

Note we expect $3 \times 2 = 6$ total possibilities

For $j = 3/2$ there are 4 $m_j$

For $j = 1/2$ there are 2 $m_j$

3) Shell model

Putting the above information together.

The complete nuclear solution can be well described by a 3D particle in a finite potential well.

a) The finite potential has approximately equally spaced energy levels.

b) The 3D nature of the problem leads spherical harmonic wave function solutions for the angular wave equation with s, p, d … orbitals as in the atomic system

c) The nuclear wave functions are located within the nucleus due to the strong attaching and different orbitals can see large differences in nuclear potential due to “charge” shielding effects leading to different potential energies and a complex ordering between $n$ and $l$ states.

d) The radial wave equation is different so the relationship between $n$ and $l$ form the separation of variables is different. There are many more $l$ for any given $n$.

e) Spin-orbit coupling is very large and leads to splittings that are sometimes larger than the $n$ and $l$ energy levels.
f) proton energy levels are generally higher than neutron levels due to proton-proton electrostatic repulsion and will lead to some neutrons orbitals that are next in the sequence being filled before previous proton levels are complete. Initially the change in ordering is partial with a neutron level filling up and the next level starting before a proton level is complete. Eventually whole levels are reversed.

The ordering of the proton levels can also be changed by the electrostatic potential and it’s effect on the total potential. This happens more in higher levels after more protons are present.

Note the “shells” start generally start with a new l quantum number in the n=1 series. Full nuclear shells tend to show the same stability that full atomic shells do. As in the atomic case the nucleon with magic number +1 will be less tightly bound.
Comparison of neutron and proton energy levels. Protons left and neutrons right. This diagram uses a shell numbering rather than a “n”. Note that 17 F is radioactive.
4) Aside: nuclear – electron total angular momentum coupling
The electron orbitals can interact with the total nuclear angular momentum. The nucleus will have a total nuclear angular momentum, typically called nuclear spin since we are considering the nucleus as one object. This is designated I and is the sum of all the nucleon spins and orbital angular momentums. Combining this with an electron total angular momentum J give $F = I + J$. The electron energy levels can split into $2J+1$ different F levels. The change in energy is proportional to the nuclear magneton, which characterizes the magnetic field of the nucleon at electron radial distances and is therefore quite small. These splitting’s are known as the hyperfine structure.