Physics 208, Lecture 6

Today’s Topics

- Capacitance (Ch. 26.1-3)
  - Capacitors and Capacitance
  - Calculating Capacitance for parallel-plate, cylindrical, spherical capacitors.
  - Combinations of capacitors

- Hope you have previewed!
About Exam 1

- When and where
  - Tuesday Oct. 2\textsuperscript{nd} 5:30-7:00 pm
  - (room to be announced)

- Format
  - Closed book
  - One 8x11 formula sheet allowed, must be self prepared, no photo copying/download-printing of solutions, lecture slides, etc.
  - 20 multiple choice questions
  - Bring a calculator (but no computer). Only basic calculation functionality can be used.
  - Bring a B2 pencil for Scantron.

- Special requests:
  - Have to be approved. Deadline is 12pm tomorrow (Sep 21\textsuperscript{st}.)
  - All specially arranged tests (e.g. those at alternative time) are held in our 202 labs. (for approved requests only)
Chapters Covered

- Chapter 23: Electric Fields
- Chapter 24: Gauss’ s Law
- Chapter 25: Electric Potential
- Chapter 26: Capacitance

I will not endorse past/sample exams as they are usually not representative. Often those can be misleading.

I will use next Thursday’s lecture to review for the test. (and will show a few sample test questions to help you get familiar with the test style)
Review: Electric Potential Difference

- Electric Potential Energy: \( q \) In a Generic E. Field

\[
\Delta U = U_B - U_A = -q \int_A^B \mathbf{E} \cdot d\mathbf{s} = q \Delta V
\]

- Electric Potential Difference

\[
\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = V_B - V_A
\]
Exercise: Parallel Plates

- Find the potential difference between the two large conductor plates of area $A$ and separation $d$

Answer

$\Delta V = \frac{Qd}{\varepsilon_0 A}$

Note: $\Delta V$ is proportional to $Q$

Realistic case
Understand a Battery (I)

- What is an 1.5V battery?

- Chemical process maintains a charge distribution, such that $V_+-V_- = 1.5V$, regardless of shape.

- Electric energy is stored in the E field.

Chemical process $\rightarrow$ charge dist. $\rightarrow$ E Field $\rightarrow$ $\Delta V=1.5V$
Understand a Battery (II) In-Use

- When connected to a load

- Electron flow from negative side to positive side.
- In the process, the 1.5V potential Diff. is maintained.
- Kinetic energy acquired by each electron 1.5 eV.
- This energy is converted into heat, light etc.
- Chemical energy $\rightarrow$ electric potential energy $\rightarrow$ load
Capacitors

- A generic capacitor:

- Unlike battery, capacitors are passive devices.
- Capacitor are very useful devices:
  - Timing control, noise filters, energy buffer, frequency generator/selector/filter, sensors, memories...

\[ \Delta V \propto Q \]
Demo: Charging A Pair of Parallel Conductors

Uncharged

Charging

\[ \Delta V = V_+ - V_- \]
Capacitance

- $\Delta V \propto Q \rightarrow Q = C \Delta V \rightarrow C$ is called capacitance
- $C = \frac{Q}{\Delta V}$: amount of charge per unit of potential diff.
  - Unit: Farad (F) = 1 Coulomb/Volt
  - Parallel-plate: $C = \varepsilon_0 \frac{A}{d}$
  - Cylindrical and Spherical: see examples in text

- **Cylindrical:**
  \[
  C = \frac{\ell}{2k_e \ln(b/a)}
  \]

- **Spherical:**
  \[
  C = \frac{ab}{k_e (b - a)}
  \]
Combinations of Capacitors In Series

Charge conservation: \( Q_1 = Q_2 (=Q) \)

\[
\begin{align*}
C_1 \Delta V_1 &= Q \\
C_2 \Delta V_2 &= Q \\
\end{align*}
\]

Effective Capacitance
\[
C = \frac{Q}{\Delta V} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}
\]

\[
1/C_{\text{series}} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots
\]

Note: \( C_{\text{series}} \) always < \( C_i \)
Combinations of Capacitors In Parallel

\[ C_1 \Delta V_1 = Q_1 \]
\[ C_2 \Delta V_2 = Q_2 \]

\[ \Delta V_1 = \Delta V_2 = \Delta V \]
(why?)

Effective Capacitance
\[ C = Q / \Delta V \rightarrow C = C_1 + C_2 \]

\[ C_{\text{parallel}} = C_1 + C_2 + C_3 + \ldots \]
Note: \( C_{\text{parallel}} \) always > \( C_i \)
Quick Quiz/exercise: Combination of Capacitors

- What is the effective capacitance for this combination?  
  \((C_1=1\mu F, C_2=2\mu F, C_3=3\mu F)\)
  1. \(C=6\mu F\)
  2. \(C=3\mu F\)
  3. \(C=1.5\mu F\)
  4. None of above

\[\begin{array}{c}
| C_1 \quad C_2 \quad C_3 |
\end{array}\]
Charging A Capacitor

Two things happen while charging:
- positive work done to system
- $\Delta V$ increasing

- Electric potential energy gained:

$$U = \int du = \int (\Delta V) (dq) = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$$

- After charging the capacitor stores potential energy:

$$U = \frac{1}{2} \frac{Q^2}{C}$$
Discharging A Capacitor

Two things happen while discharging:
• electric field does positive work
• ΔV decreasing

Potential energy released:

\[ U = \int dU = - \int \Delta V \, dq = \int_0^0 \frac{-q}{C} \, dq = \frac{1}{2} Q^2 / C \]

The originally charged capacitor has potential energy:
\[ U = \frac{1}{2} Q^2 / C \]