Physics 202, Lecture 5

Today’s Topics

- Electric Potential Energy
- Electric Potential
  - Electric Potential For Various Charge Distributions
    - Point charges
    - Continuous charges
      e.g. Uniform Ring, Sphere, Shell

- Expected from preview:
  - $E \leftrightarrow V$ relationship, equipotential lines, electrostatic equilibrium of conductors...
  - Electric potential for a charge distribution...
Review: Electric Potential Energy Between Point Charges

- Electric energy between two point charges:
  \[ U = U - U_\infty = K_e \frac{q_0 q}{r} \]
  - \( U \) is a scalar quantity
  - \( U = 0 \) at \( r = \infty \) (convenient convention)
  - \( U \) can be positive or negative
    - \( + \): between like-sign charges
    - \( - \): between opposite charges
  - SI unit: Joule (J)

- Electric potential energy for system of multiple charges/charge distributions:
  \[ U = \Sigma \text{of all combination of pairs.} \]
Example: Three Charge system

- What is the work required to assemble the three charge system as shown? \((q_1=q_2=q_3=Q)\)
  Answer: \(k_e \frac{3Q^2}{a}\) (see board)

- Quiz: What if \(q_1=q_2=Q\) but \(q_3=-Q\)?
  Answer: \(-k_e \frac{Q^2}{a}\)
Electric Potential Energy
For Charge In An Electric Field

- Charge q is subject an electric force in electric field $\mathbf{E}$

$$\mathbf{F} = q \mathbf{E}$$

- Work done by electric force:

$$W = \int_{i}^{f} \mathbf{F} \cdot d\mathbf{s} = q \int_{i}^{f} \mathbf{E} \cdot d\mathbf{s} = - \Delta U$$

$$\Delta U = U_{f} - U_{i} = -q \int_{i}^{f} \mathbf{E} \cdot d\mathbf{s}$$

independent of q
Electric Potential Difference

Electric Potential Energy: \( q \) In a Generic E. Field

\[
\Delta U = U_B - U_A = -q \int_A^B \mathbf{E} \cdot d\mathbf{s} = q \Delta V
\]

Electric Potential Difference

\[
\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = V_B - V_A
\]
Properties of Electric Potential Difference

- It is defined upon the fact that the electric force is a conservative force.
- It is associated to the source field only and is independent of test charge.
- It has a unit: \( \text{J/C} \equiv \text{Volt (V)} \)
- It is commonly called as just Potential, but it is meaningful only as potential difference \( V_B - V_A \).
- Usually a convenient point (remote, earth..) is chosen as “ground” \( \Rightarrow \Delta V = V - (V_A \equiv 0) = V \)
- It is a scalar quantity. (No vector operation necessary!)
- \( \Delta U = q \Delta V \)
Exercise/Quiz: Potential In Uniform E. Field

- For a uniform electric field shown. Consider point “A” in the field, let’s set potential $V_A=0$.

Wait: can we do that (set $V_A=0$ at wish)? (Trivial quiz)

A: Yes, we can always choose any one (and only one) reference point to have $V=0$.

B: No, we can only set infinity to have $V=0$. 

(a)
Quiz: Potential In Uniform E. Field (2)

- Now consider point B in the same field. Still set $V_A = 0$
- What is the electric potential at point B?

1: $V_B = 0$

2: $V_B = -Ed$

3: $V_B = +Ed$

4: Not enough information to determine.

What about $V_C$, $V_D$, and $V_G$?

Answer: $V_C = V_B = -Ed$; $V_D = V_A = 0$, $V_G = -1/2 Ed$
Quiz: Potential Energy In Uniform E. Field

- Now we know: $V_D=V_A=0$, $V_C=V_B=-Ed$, $V_G=-1/2Ed$
- If a charge $+q$ is placed at B, what is the potential energy $U_B$?
  Answer: $U_B=-qEd$
- If a charge $-q$ is placed at B, what is the potential energy $U_B$?
  Answer: $U_B=+qEd$

- If a negative charge $-q$ is initially at rest at G, will it move to A or B?
  Answer: A

- What is the kinetic energy when it reaches A?
  Answer: $E_k=1/2 qEd$
Field lines always point towards lower electric potential.

Field lines and equal-potential lines are always at a normal angle.

In an electric field:
- a +q is always subject a force in the same direction of field line. (i.e. towards lower V)
- a -q is always subject a force in the opposite direction of field line. (i.e. towards higher V)
Exercise: E. Potential and Point Charges

- In the configuration shown,
  - Find the potential difference \( V_B - V_A \)

Answer:

\[ V_B - V_A = k_e \left( \frac{q}{r_B} - \frac{q}{r_A} \right) \]

(Exercise with your TA)
Visualization of Electric Potential
Equipotential Lines

(a)

(b)

(c)
More Examples: Uniformly Charged Spherical Shell

- For uniformly charged spherical shell.

Again, use:

\[ \Delta V = - \int_{A}^{B} \mathbf{E} \cdot d\mathbf{s} = V_B - V_A \]

Tip:
- \( V \) is the same inside \( E=0 \) region

\[ E = 0 \quad r < R \]

\[ E_r = \frac{k_e Q}{r^2} \quad r > R \]
More Examples: Uniformly Charged Sphere

Show that for a uniformly charged sphere, the electric potential is:

\[ V_0 = \frac{3k_eQ}{2R} \]

\[ V_D = \frac{k_eQ}{2R} \left( 3 - \frac{r^2}{R^2} \right) \]

\[ V_B = \frac{k_eQ}{r} \]

Exercise with TA

\[ \frac{2}{3} V_0 \]

\[ R \]

\[ r \]

Hint: It is more convenient to use:

\[ \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = V_B - V_A \]

since from Gauss’ law:

\[ E_r = \frac{k_eQ}{r^2} \quad r > R \]

\[ E_r = \frac{k_eQr}{R^3} \quad r < R \]
Electric Potential For Continuous Charge Distribution

- For finite charge distribution, it is common to set $V=0$ at infinite.

$$V = k_e \int \frac{dq}{r}$$

- If the charge distribution is known, $V$ can be calculated simply by scalar integral.

$$dV = k_e \frac{dq}{r} \quad (V=0 \ @ \ \infty)$$
Example: Uniformly Charged Ring

For a uniformly charged ring, show that the potential along the central axis is

\[ V = \frac{k_e Q}{\sqrt{x^2 + a^2}} \]

Solution

\[ V = \int \frac{k_e dq}{r} = \int \frac{k_e dq}{\sqrt{x^2 + a^2}} = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = Q \]
Calculate Electric Field From The Electric Potential

- Three ways to calculate the electric field
  - Superposition $\mathbf{E} = \Sigma \mathbf{E}_i$
  - Gauss’ s Law
  - From the gradient of electric potential
    - Formulism

$$\Delta V = - \int_{A}^{B} \mathbf{E} \cdot ds$$

$$dV = - \mathbf{E} \cdot ds = - E_x dx - E_y dy - E_z dz$$

$$E_x = - \frac{\partial V}{\partial x}, \quad E_y = - \frac{\partial V}{\partial y}, \quad E_z = - \frac{\partial V}{\partial z} \quad or \quad \mathbf{E} = - \nabla \cdot V$$
Uniformly Charged Ring: Electric Field

- Find the electric field along the central axis.

**Approach 1: Superposition.** (Example 23.7 in text)

\[
dE_x = dE \cos \theta = \frac{k_e dq}{r^2} \frac{x}{r}
\]

\[
E_x = \int dE_x = \frac{k_e x Q}{r^3} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}}
\]

\[
E_\perp = 0 \text{ due to symmetry}
\]

**Approach 2: derivative of potential**

\[
E_x = - \frac{\partial V}{\partial x} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}}
\]