Physics 202, Lecture 4

Today’s Topics

- More Examples on Gauss’s Law
- Conductors in Electrostatic Equilibrium (Ch. 24.4)
- Electric Potential (Ch. 25-Part I)
  - Electric Potential Energy & Electric Potential
  - Electric Potential And Electric Field

- Expected from preview:
  - Conductors
  - Conservative force, electric energy, ..
Review: The Gauss’s Law

- The Gauss’s Law: The net electric flux through any closed surface (also called Gaussian surface) equals to the total charge enclosed inside the closed surface divided by the free space permittivity.

\[ \Phi_E \equiv \oint E \cdot dA = \frac{\sum q_{\text{in}}}{\varepsilon_0} \]

- \( q_{\text{in}} \): all q’s enclosed regardless of positions
- \( \varepsilon_0 \): permittivity constant
- \( 1/4\pi\varepsilon_0 = K_e \)
- Gaussian surface (any shape)
Gauss’s Law and Spherical Symmetry

- For a uniformly charged sphere (solid, or spherical shell), consider a concentric Gaussian surface.

Due to symmetry:
- $E$ must be radial (normal to Gaussian surface).
- $E$ has the same strength in all direction.

\[
\oint \vec{E} \cdot d\vec{A} = E \ A = 4\pi r^2 E
\]

\[
E = \frac{q_{\text{inclosed}}}{4\pi r^2 \varepsilon_0}
\]

Question (challenge your TA):
How to get $q_{\text{inclosed}}$ if the Gaussian surface is inside the sphere?
Gauss’s Law and Cylindrical Symmetry

- For a **uniformly charged** cylinder (or line) of **infinitely** long, consider a cylindrical Gaussian surface concentric to the cylinder.

Due to symmetry:
- E must be radial (normal to cylinder surface)
- E has the same strength in all direction at the same radius r.

\[ \oint \vec{E} \cdot d\vec{A} = E \ A_{side} = 2\pi r \ell E \quad (\ell: \text{ length}) \]

\[ E = \frac{q / \ell}{2\pi r \varepsilon_0} \]
Gauss’s Law and Planar Symmetry

- For a **uniformly charged** plate (sheet) of **infinitely large**, consider a Gaussian surface that forms a rectangular box that cuts the planar plate in the middle.

Due to symmetry:
- **E** must be normal to the plate surface.
- **E** has the same strength at all points.

\[
\oint \vec{E} \cdot d\vec{A} = 2E \ A
\]

\[
E = \frac{q_{\text{inclosed}}}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}
\]

\[
(\sigma \equiv \frac{q}{A} : \text{surface charge density})
\]
Three Common Symmetric Cases

- **Spherical**
  - (point Q, uniform sphere, shell)

- **Cylindrical**
  - (infinite line/cylinder of Q )

- **Planar**
  - (infinite sheet of Q )

The above symmetric settings give very predictable $E$

$\rightarrow$ **Direction:** Normal to surfaces of same symmetry

$\rightarrow$ **Magnitude:** Same across surface (of same symmetry)

$$\int \vec{E} \cdot d\vec{A} = EA$$
Conductors And Electrostatic Equilibrium

- Conductors: Total charge are initially balanced (=0) but negative charge (electrons) are able to move freely inside its body.
  - capable of redistributing charges when subject to an external electric field.
- Charge redistribution $\rightarrow$ eventually electrostatic equilibrium.

Initial $\rightarrow$ transient, $<10^{-8}$s $\rightarrow$ equilibrium (right after $E$ applied)
Properties of Electrostatic Equilibrium

- Once in electrostatic equilibrium
  - The electric field is always zero inside the conductor
  - E field on the surface of conductor is always normal to the surface,
    - and has a magnitude of $\sigma/\varepsilon_0$ (Show using Gauss’ s law)
  - All net charges reside on the surface of conductor (i.e. no net charge inside the body of a conductor).
  
  - The electric field is also zero inside any empty cavity within the conductor.
  - Electric potential is the same over the whole conductor (Ch. 25)

The above properties are valid regardless of the shape and the total charge of the conductors!
Quiz 1: Charge Distribution On Conductors

- The total charge on this conductor sphere is 5q, how is the charge distributed?
  1. Evenly distributed throughout the body
  2. $Q_{\text{surface}} = 5q$, $Q_{\text{body}} = 0$
  3. None of above

Note: Regardless shapes, charge resides only on the surface of a conductor
Quiz/Exercise 2: Charge Distribution On Conductors

The total charge on this conductor shell is $5q$. How is the charge distributed? ($R_{\text{outer}} = 2R_{\text{inner}}$)

- $Q_{\text{Inner surface}} = 2.5q$, $Q_{\text{Outer surface}} = 2.5q$, $Q_{\text{body}} = 0$
- $Q_{\text{Inner surface}} = q$, $Q_{\text{Outer surface}} = 4q$, $Q_{\text{body}} = 0$
- $Q_{\text{Inner surface}} = 0$, $Q_{\text{Outer surface}} = 5q$, $Q_{\text{body}} = 0$

Note: Regardless shape, charge resides only on outer surface of a conductor if no charge inside cavity (Aslo $E_{\text{inside}} = 0$).
Quiz/Exercise 3: Charge Distribution On Conductors

The total charge on this conductor shell is $+5q$. A point charge of $+q$ is placed at the center. How is the charge distributed?

(Diameter $= 2R$)

- $Q_{\text{Inner surface}} = -q$, $Q_{\text{Outer surface}} = 6q$, $Q_{\text{body}} = 0$
- $Q_{\text{Inner surface}} = q$, $Q_{\text{Outer surface}} = 4q$, $Q_{\text{body}} = 0$
- $Q_{\text{Inner surface}} = 0$, $Q_{\text{Outer surface}} = 5q$, $Q_{\text{body}} = 0$
The total charge on this shell is initially $+5q$. A point charge of $+q$ is placed at the center and the shell is now grounded. How is the charge distributed? ($R_{outer}=2R_{inner}$)

- $Q_{Inner\_surface} = -q$, $Q_{Outer\_surface} = 6q$, $Q_{body} = 0$
- $Q_{Inner\_surface} = q$, $Q_{Outer\_surface} = 4q$, $Q_{body} = 0$
- $Q_{Inner\_surface} = -q$, $Q_{Outer\_surface} = 0$, $Q_{body} = 0$
- $Q=0$ everywhere.
Potential Energy (Phy201 Review)

- Ch-8: path independent work $\Rightarrow$ conservative force.
- e.g. Gravitational Force is a conservative force (Ch-13):

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

$$W = \int_{path} \vec{F} \cdot ds = Gm_1 m_2 \frac{r_f}{r_f} - Gm_1 m_2 \frac{r_i}{r_i}$$

Gravitational Potential energy:

$$U = - \frac{Gm_1 m_2}{r}$$

Electric Force:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

Electric Potential Energy

$$U = \frac{k_e q_1 q_2}{r}$$
Electric Potential Energy

Electric energy between two point charges:

\[ U = U - U_\infty = K_e \frac{q_0 q}{r} \]

- U is a scalar quantity
- U = 0 @ r = \infty (convenient convention)
- U can be positive or negative
  - +: between like-sign charges
  - -: between opposite charges
- SI unit: Joule (J)

Electric potential energy for system of multiple charges/charge distributions:

\[ U = \sum \text{of all combination of pairs.} \]

Integral if continuous distribution