Physics 202, Lecture 3

Today’s Topics

- Calculate Electric Field With Superposition (Direct Sum/Integral of Coulomb’s Law)
- Calculate Electric Field With Gauss’s Law
- Gauss’s Law
- Examples

Expected from preview:
- Calculate E with continuous charge.
- Surface, closed surface, surface integral, flux, the Gauss’s Law.

Review: Electric Field and Electric Force

Electric Field is a form of matter. It carries energy (later in the semester).

How to Calculate Electric Field?

- Single point-like:
  \[ \vec{E} = k_e \frac{q}{r^2} \hat{r} \]

- Multiple charges:
  (superposition principle)
  \[ \vec{E} = k_e \sum \frac{q_i}{r_i^2} \hat{r}_i \]

- Continuous Charge Distribution:
  \[ \vec{E} = k_e \lim_{\Delta q \to 0} \sum \frac{\Delta q}{r^2} \hat{r} - \kappa \int \frac{dq}{r^2} \hat{r} \]

Note: For now, we assume charges are not moving. (electrostatic)

Example: Charged Rod

- A uniformly charged rod of length \( L \) has a total charge \( Q \), find the electric field:
  - at point A → answer: \( E_x = -k_e Q/(a(L+a)) \), \( E_y = 0 \) (see board)
  - at point B → answer: \( E_y = 2k_e Q/(Lb) \sin \theta_0 \), \( E_x = 0 \) \( \tan \theta_0 = \frac{1}{2}L/b \) (see board, show method only)
  - at an arbitrary point C. (see board, conceptual only).
**Example: Uniformly Charged Sphere**

- A uniformly charged sphere has a radius $a$ and total charge $Q$, find the electric field outside and inside the sphere.
- Solution:

Don’t take notes of my solution: I AM FOOLING ARUOND!
It is quite complicated with the superposition method!
→ Gauss’s Law to the rescue!

**Electric Flux**

- The electric flux through a surface element is defined as the dot product of the electric field and the surface area vector:
  $$\Delta \Phi = E \cdot \Delta A = E \Delta A \cos \theta$$

- The net electric flux through a closed surface
  $$\Phi_E = \int E \cdot dA$$

**The Gauss’s Law**

- The Gauss’s Law: The net electric flux through any closed surface (also called Gaussian surface) equals the total charge enclosed inside the closed surface divided by the free space permittivity.

$$\Phi_E = \int E \cdot dA = \sum q_{i}$$

- $q_i$: all $q$’s enclosed regardless of positions

$$\varepsilon_0$$: permittivity constant

**Trivia Quiz 1**

- Compare electric fluxes through closed surfaces $S_1, S_2, S_3$:
  1. $\Phi_{S_1} > \Phi_{S_2} > \Phi_{S_3}$
  2. $\Phi_{S_1} = \Phi_{S_2} = \Phi_{S_3}$
  3. $\Phi_{S_1} < \Phi_{S_2} < \Phi_{S_3}$
Trivia Quiz 2

- What is the electric flux through closed surface S?
  1. $\Phi = 0$
  2. $\Phi = \frac{q_1 + q_2 + q_3 + q_4 + q_5}{\varepsilon_0}$
  3. $\Phi = \frac{(q_1 + q_2 + q_3)}{\varepsilon_0}$

Uniformly Charged Sphere Again

- Solution using Gauss’ Law: $\mathbf{E} \cdot d\mathbf{A} = \frac{1}{E_0} q_{in}$

The setting is highly symmetrical
- Gaussian surface will be concentric sphere of radius $r$.

How to evaluate $\int \mathbf{E} \cdot d\mathbf{A}$?

Note the symmetry:
- Direction of $E$: Radial
- Magnitude of $E$: Same in all direction

$\int \mathbf{E} \cdot d\mathbf{A} = \oint E dA = E \oint dA = E \cdot A = 4\pi r^2 E$

Uniformly Charged Sphere: Details

- $r > a$: Outside the Sphere
- $r < a$: Inside the Sphere

$\int E \cdot d\mathbf{A} = 4\pi \varepsilon_0 E = \frac{1}{\varepsilon_0} q_{in}$

$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$

where: $q_{in} = \begin{cases} \frac{Q}{r^2} & \text{if } r > a \\ \frac{Q}{a^2} & \text{if } r = a \end{cases}$

Uniform Charge Sphere: Final Solution

Note:
- This has the same form as the point charge
Procedure to Use Gauss’s Law

\[ \oint E \cdot dA = \frac{q_{\text{enc}}}{\varepsilon_0} \]

- **General principle:** Gauss’s law is valid for any charge distributions, but practically it is useful only in limited situations where the charge distribution is highly symmetric.

- **Procedure:**
  1. Draw a Gaussian surface passing the field point concerned. Observe symmetry so that the surface integral is trivial.
     - Direction of \( E \): Either perpendicular or parallel to the surface.
     - Magnitude of \( E \): The same (or be zero) on the surfaces.
  2. Evaluate the surface integral using arguments of symmetry. And Equate the surface integral to \( \frac{q_{\text{enc}}}{\varepsilon_0} \) and solve for \( E \).

Three Common Symmetric Cases

- **Spherical**
  - (point \( Q \), uniform sphere, shell)
- **Cylindrical**
  - (infinite line/cylinder of \( Q \))
- **Planar**
  - (infinite sheet of \( Q \))

The above symmetric settings give very predictable \( E \)
- **Direction:** Normal to surfaces of same symmetry
- **Magnitude:** Same across surface (of same symmetry)

Another Example: Thin Spherical Shell

- Find the \( E \) field inside/outside a uniformly charged thin sphere.

Solution: Exercise with your TAs.

Typical Electric Field Obtainable By Gauss’s Law

<table>
<thead>
<tr>
<th>Charge Distribution</th>
<th>Electric Field</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulating sphere of radius ( R ), contains charge density ( \rho ) and total charge ( Q )</td>
<td>( E = \frac{\rho R^2}{4 \varepsilon_0} ), ( r &gt; R )</td>
<td>Inside the sphere</td>
</tr>
<tr>
<td>Thin spherical shell of radius ( R ) and total charge ( Q )</td>
<td>( E = \frac{Q}{2 \pi \varepsilon_0 R} ), ( r &lt; R )</td>
<td>Inside the shell</td>
</tr>
<tr>
<td>Line charge of infinite length and charge per unit length ( \lambda )</td>
<td>( E = \frac{\lambda}{2 \pi \varepsilon_0 r} ), ( r &gt; R )</td>
<td>Outside the line</td>
</tr>
<tr>
<td>Infinite charged plane having surface charge density ( \sigma )</td>
<td>( E = \frac{\sigma}{2 \varepsilon_0} )</td>
<td>Everywhere outside the plane</td>
</tr>
<tr>
<td>Conductor facing surface charge density ( \sigma )</td>
<td>( E = \frac{\sigma}{\varepsilon_0} )</td>
<td>Just outside the conductor</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inside the conductor</td>
</tr>
</tbody>
</table>

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One More Exercise On Gauss’ s Law

Two charges +2Q and –Q are placed at locations shown. Find the electric field at point P.

Solution:

1. Draw a Gaussian surface passing P
2. Apply Gauss’ s law:

\[ \oint E \cdot dA = \frac{q_{in}}{\varepsilon_0} \]

3. \( q_{in} = +2Q + (-Q) = Q \)
4. Surface integral:

\[ \oint E \cdot dA = \frac{4\pi Q}{r^2} \]

5. \( E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \)

Is this correct? No! Which step is wrong? step 4