Today’s Topics

- Diffraction
  - Single-slit Diffraction
  - Reminder of Two-slit Interference
  - Double-Slit Diffraction
  - Diffraction on Circular Apertures
  - The Rayleigh Criterion
- Polarization
Reminder:
Condition for Ray Approximation

- When the wavelength of the light is much smaller than the size of the optical objects it encounters, it can be treated as (colored) rays.

Ray approximation is valid when $\lambda \ll d$

Ray approximation is not valid near the gap when $\lambda \sim d$. OK elsewhere.
Reminder: Single-Slit Interference (Single-Slit Diffraction)

If lights were just rays
Single-Slit Diffraction Pattern Explained

- The slit is not a point source → Interference

\[ E_p = \sum E_i \]

\[ = \sum \left( \frac{\Delta y}{D} E_0 \right) \sin(\omega t + \frac{2\pi}{\lambda} y \sin \theta) \]

\[ = \int_{-D/2}^{D/2} \frac{E_0}{a} \sin(\omega t + \frac{2\pi}{\lambda} y \sin \theta) dy \]

\[ = \frac{2 E_0}{D} \sin(\frac{2\pi}{\lambda} \frac{D}{2} \sin \theta) \sin(\omega t) \]

\[ I = I_0 \left[ \frac{\sin(\frac{\beta}{2})}{\frac{\beta}{2}} \right]^2 \]

\[ \beta \equiv \frac{2\pi}{\lambda} D \sin \theta \]

The text also offers a derivation using phasors. Not to be examined but please read.
Where Are the Dark Fringes?

The dark fringes occur at:

\[ I=0 \rightarrow \sin(\beta/2)=0 \rightarrow \sin\theta_{dark}=m\lambda/D, \ m=\pm 1, \pm 2, \pm 3,... \]

Central bright dot width \( \Delta\theta = 2\lambda/D \)

First dark fringes at \( \pm \lambda/D \)
Lights As Rays?

- Due to diffraction, light beam of finite size can never travel as perfect straight rays!

- Numerical example: Estimate the size of laser beam on screen ($\lambda \sim 600$ nm, $L = 1$ m)
  
  \[
  a = 1\text{ cm}, \quad \theta \sim \frac{\lambda}{a} = 6 \times 10^{-5}, \quad a' \sim 1.01\text{ cm} \quad \rightarrow \quad +1\%
  \]
  
  \[
  a = 2\text{ mm}, \quad \theta \sim \frac{\lambda}{a} = 3 \times 10^{-4}, \quad a' \sim 2\text{ mm} + 0.6\text{ mm} \quad \rightarrow \quad +30\%
  \]
  
  \[
  a = 1\text{ mm}, \quad \theta \sim \frac{\lambda}{a} = 6 \times 10^{-4}, \quad a' \sim 1\text{ mm} + 1\text{ mm} \quad \rightarrow \quad +100\%
  \]
  
  \[
  a = 0.1\text{ mm}, \quad \theta \sim \frac{\lambda}{a} = 6 \times 10^{-3}, \quad a' \sim 0.1\text{ mm} + 1.2\text{ cm} \quad \rightarrow \quad +12000\%
  \]
Reminder: Two-slit Interference

Reminder: Two-slit Interference

Separation between minima = \( \frac{\lambda}{d} \)

\[ I = I_o \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \]
Two-slit Diffraction

\[ I = I_0 \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2 \]

\[ \beta \equiv \frac{2\pi}{\lambda} D \sin \theta \]

\[ I = I_o \cos^2 \left( \frac{\delta}{2} \right) \]

\[ \delta = \frac{2\pi}{\lambda} d \sin \theta \]

(a) Diffraction factor, \((\sin^2 \beta/2)/(\beta/2)^2\) vs. \(\theta\)

(b) Interference factor, \(\cos^2 \frac{\delta}{2}\) vs. \(\theta\)

(c) Intensity, \(I_0\) vs. \(\theta\)
Diffraction on Circular Apertures

- Light through apertures will produce diffractive patterns depending on their shape. For circular apertures the diffractive pattern is made of concentric rings.

First dark ring at $\theta = \frac{1.22\lambda}{D}$. 

![Diagram showing intensity versus angle for circular aperture diffraction with first dark ring indicated.](image-url)
Resolution of Single-slit and Circular Apparatus

two separate beams

each smeared due to diffraction

Rayleigh’s Criterion

Separable  Minimally separable  Not separable

- Single slit: $\theta_{\text{min}} = \lambda/D$
- Circular Aperture: $\theta_{\text{min}} = 1.22 \lambda/D$
Example: Watching HDTV

- Quiz: Human eye has a typical pupil diameter of about 5 mm.

What is the minimum distance between two red ($\lambda = 700\text{nm}$) dots human eye can separate at 3 meters?

\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \frac{700\text{nm}}{5\text{mm}} \approx 1.7 \times 10^{-4} \text{ Rad}
\]

\[
\Delta S_{\text{min}} \approx \theta_{\text{min}} \times L \approx 1.7 \times 10^{-4} \times 3000 \text{ mm} = 0.51 \text{ mm}
\]

- Compare the above resolution to the pixel spacing of a 32” HDTV (720p or 1080p), what conclusion can you get?

<table>
<thead>
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<th>HDTV Geometric Parameters</th>
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<td>Screen Size (inch)</td>
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Polarization of Light Waves

- Linear polarization

- "Un-polarized" = equal mixture of all polarizing directions
Certain material (e.g. Calcite) allow only lights with polarization in parallel to its transmission axis to pass through. (E.H. Land, 1938)

Light passing through two polarizers: Malus’ s Law

\[ I_0 \propto E_0^2 \]

\[ I = I_0 \cos^2 \theta \]
Quick Quiz

Consider a beam of un-polarized light passing through a polarizer. If the intensity of the un-polarized light is $I$, what is the intensity of the beam after the polarizer?

- $I$
- $\frac{1}{2}I$
- $\frac{1}{4}I$
- None of above.

**Hint:** You may use Malus’ law, or Energy Conservation.

**Solution:**

- Use Malus’ Law:
  \[ I_{after} = \int_0^{\pi} I \cos^2 \theta \, d\theta = \frac{1}{2} I \]
- Use energy conservation. (Share your thoughts, please).
Demo: Malus’ Law

\[ I = I_0 \cos^2 \theta \]

\( \theta = 0^\circ \) \quad \theta = \sim 30^\circ \quad \theta = 90^\circ 

more reading: Polarization by reflection, double refraction, scattering,...
Physics 201 and 202

- Light and Optics
- Electro-Magnetism
- Thermodynamics
- Oscillation and Waves
- Classical Mechanics
- Classical

Cosmology
Sub-Sub-Atomic: Elementary Particles
Sub-Atomic: Nuclear Physics
Many-Atoms: Molecules, solids
Atomic Structure
Quantum Theory
Relativity
Modern