Physics 202, Lecture 28

Today’s Topics

- Review: Wave Optics: Interference
- Thin Film Interference
  - Change of Phase at Boundaries
  - Exercise on Thin Film Interference
  - Exercise on Anti-Reflective Coating
- Diffraction (Ch. 38.)
  - Single Slit interference
Review: Interference of Light Waves

➢ Two light waves with same color and amplitude.

\[ E_1 = E_0 \sin(\omega t - kx_1 + \phi_{10}) = E_0 \sin(\omega t + \phi_1) \]
\[ E_2 = E_0 \sin(\omega t - kx_2 + \phi_{20}) = E_0 \sin(\omega t + \phi_2) \]

\[ \rightarrow E = E_1 + E_2 = 2E_0 \cos(\Delta \phi/2) \sin(\omega t + \phi) \]

\[ \rightarrow \text{Resulting amplitude: } E_{\text{max}} = 2E_0 \cos(\Delta \phi/2) \]

- Constructive interference: \( \Delta \phi = 0, 2\pi, 4\pi, \ldots \)
  \( E_{\text{max}} = 2E_0 \)
- Destructive interference: \( \Delta \phi = \pi, 3\pi, 5\pi, \ldots \)
  \( E_{\text{max}} = 0 \)

\[ \rightarrow \text{It all depends on } \Delta \phi ! \]
Review: How to Calculate $\Delta \phi$?
Path Length And Path Length Difference

For two interfering waves coming through different paths the phase difference:

$$\Delta \phi = \Delta \phi_{\text{at the source}} + \Delta \phi_{\text{due to path}} + \Delta \phi_{\text{phase transition}}$$

- $\Delta \phi = 0$ in many cases
- $\Delta \phi = k(r_2 - r_1) = \frac{2\pi}{\lambda} (r_2 - r_1)$
- See later

where $r_1$ and $r_2$ are path lengths, $\Delta r = (r_1 - r_2)$ is called path length difference.

Recall phy201
Possible Phase Change of $180^\circ$ For Reflected Light

- When a light traveling in medium 1 of $n_1$ is reaches at a boundary with medium 2 of $n_2$:
  - The reflected light has a $180^\circ(\pi)$ phase shift if $n_1 < n_2$
  - There is no phase change for reflected light if $n_1 > n_2$
  - In any case, no phase shift for refracted light

For $n_1 < n_2$:
- $180^\circ(\pi)$ phase shift

For $n_1 > n_2$:
- $0^\circ$ phase shift

(a) 180° phase change
(b) No phase change
Thin Film Interference

- Thin film splits light \( \rightarrow \) split lights then interfere

\[ \theta \approx 0^\circ \]

\[ \text{lights 1,2 interfere} \]

\[ \text{phase change } \pi \text{ for light 1} \]

\[ \Delta \phi_{12} \approx \frac{2\pi}{\lambda_n} (2t) + \pi \]

\[ n \rightarrow \lambda_n = \frac{\lambda}{n} \]

\[ \text{lights 3,4 also interfere} \]

\[ \Delta \phi_{34} \approx \frac{2\pi}{\lambda_n} (2t) \]

Quiz: Constructive/destructive Conditions?
Exercise: Non Reflective Coating

- Determine the minimum thickness \( t \) of SiO coating so a light of 550nm is non-reflective at the surface.

Solution (see board):
Non “reflective”
→ 1 and 2 cancel each other (destructive interference)

\[ \Delta \phi_{12} = \frac{2\pi}{\lambda_n} \]
\[ 2t + 0^\circ = \pi \]

→ \( t = \frac{\lambda_n}{4} = \frac{\lambda}{4n} = 94.8 \text{ nm} \)

Note \( t \) is \( \lambda \) dependent.
Exercise: Pro-Reflective Coating

- Determine the minimum thickness (t) of SiO coating so a light of 550nm is max-reflective at the surface.

Solution (see board):
Pro- “reflective”
→ 1 and 2 interference constructively

\[ \Delta \phi_{12} = \frac{2\pi}{\lambda_n} \ 2t + 0^\circ = 0, \text{ or } 2\pi \]

→ \( t = \frac{\lambda_n}{2} = \frac{\lambda}{2n} = 189.6 \text{ nm} \).

Note \( t \) is \( \lambda \) dependent.
Newton’s Rings

Demos

Testing glass for flatness
Color Separation: Make Colorful World Out Of (White) Daylight

- Color: Light with certain frequency.
- Daylight: a mixture of all colors → appears white.
- Three ways to make daylight colorful:
  - Filtering: Only one color is allowed to pass
  - Dispersion: Different colors at different refractive angles
  - Interference: Different colors get enhanced/weakened at different path-length difference, which is a function of thickness, observing angle, etc. (pictures next page)
Colorful Interference Patterns
Condition for Ray Approximation

- When the wavelength of the light is much smaller than the size of the optical objects it encounters, it can be treated as (colored) rays.

Ray approximation is valid when $\lambda \ll d$

Ray approximation is not valid near the gap when $\lambda \sim d$. OK elsewhere
Single-Slit Interference (Single-Slit Diffraction)

If lights were just rays
Single-Slit Diffraction Pattern Explained

- The slit is not a point source → Interference

\[ E_p = \sum E_i \]
\[ = \sum \left( \frac{\Delta y}{D} E_0 \right) \sin(\omega t + \frac{2\pi}{\lambda} y \sin \theta) \]
\[ = \int_{-D/2}^{D/2} \frac{E_0}{a} \sin(\omega t + \frac{2\pi}{\lambda} y \sin \theta) dy \]
\[ = \frac{2E_0}{D} \sin\left(\frac{2\pi}{\lambda} \frac{D}{2} \sin \theta\right) \sin(\omega t) \]

\[ I = I_0 \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2 \]

\[ \beta = \frac{2\pi}{\lambda} D \sin \theta \]

The text also offers a derivation using phasors.
Not to be examined but please read.
Where Are the Dark Fringes?

The dark fringes occur at:

\[ I=0 \rightarrow \sin(\beta/2)=0 \rightarrow \sin\theta_{\text{dark}}=m\lambda/D, \ m=\pm 1, \pm 2, \pm 3, \ldots \]

◊ Central bright dot width \( \Delta\theta = 2\lambda/D \)

→ First dark fringes at =± \( \lambda/D \)

\[ I = I_0 \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2 \]

\[ \beta = \frac{2\pi}{\lambda} D \sin\theta \]