Physics 202, Lecture 22

Today’s Topics

- Electromagnetic Waves (EM Waves)
- Review: Waves and Wave Equation
- Maxwell’s equation
- Propagation of $E$ and $B$
- Energy Carried by EM Wave, Poynting Vector
- Momentum Carried by EM Wave
- Spectrum of EM wave.

Maxwell Equations

\[
\oint E \cdot dA = \frac{q}{\varepsilon_0} \quad \rightarrow \text{Gauss’ Law/ Coulomb’s Law}
\]
\[
\oint B \cdot dA = 0 \quad \rightarrow \text{Gauss’ Law of Magnetism, no magnetic charge}
\]
\[
\oint E \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \rightarrow \text{Faraday’s Law}
\]
\[
\oint B \cdot d\vec{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \rightarrow \text{Ampere Maxwell Law}
\]

Also, Lotzntz force Law $\Rightarrow \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

These are the foundations of the electromagnetism

Electromagnetic Waves

- EM wave equations:
  \[
  \frac{\partial^2 E_x}{\partial x^2} = \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_z}{\partial t^2} = \frac{\partial^2 B_y}{\partial x^2} = \frac{\partial^2 B_z}{\partial t^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_x}{\partial t^2}
  \]
- Plane wave solutions:
  \[
  E = E_{max} \sin(kx - \omega t + \phi) \quad B = B_{max} \sin(kx - \omega t + \phi)
  \]
- Properties:
  - No medium is necessary.
  - $E$ and $B$ are normal to each other
  - $E$ and $B$ are in phase
  - Direction of wave is normal to both $E$ and $B$
  - Speed of EM wave: $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s}$
  - $E/B = E_{max}/B_{max} = c$
  - Transverse wave: two polarizations possible

The EM Wave

Two polarizations possible
Wavelength and Frequency

- Because of the wave equation the wavelength of an EM wave in vacuum are related by:
  \[ \lambda f = c = 3 \cdot 10^8 \text{ m/s} \]

- Example: Determine the wavelength of an EM wave of frequency 50 MHz in free space
  \[ \lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{50 \text{ MHz}} = \frac{3 \cdot 10^8 \text{ m/s}}{5 \cdot 10^7 \text{ s}^{-1}} = 6 \text{ m} \]

Energy Carried By EM Waves

- Recall: energy densities \( u_E = \frac{1}{2} \varepsilon_0 E^2 \), \( u_B = \frac{1}{2} B^2/\mu_0 \)
- For a EM wave, at any time/location,
  \[ u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 \quad \text{(using } E/B = c) \]
  \( \rightarrow \) In an electromagnetic wave, the energies carried by electric field and magnetic field are always the same.
- Total energy stored (per unit of volume):
  \[ u = u_E + u_B = \varepsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 \]
- Power transmitted per unit of area is equal to \( uc \) in the direction of wave
- Averaging over time:
  \[ u_{av} = \frac{1}{2} \varepsilon_0 E_{max}^2 = \frac{1}{2} B_{max}^2/\mu_0 \quad \text{, } u_{av}c = I \text{ (intensity)} \]

The Poynting Vector

- The rate of flow of energy in an EM wave is described by a vector, \( \vec{S} \), called the Poynting vector
- The Poynting vector is defined as
  \[ \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]
  \[ I = S_{av} \]
- Its direction is the direction of propagation
- This is time dependent
  - Its magnitude varies in time
  - Its magnitude reaches a maximum at the same instant as \( E \) and \( B \)
Momentum Carried By EM Waves

- EM waves: momentum = energy/c

辐射压力 (P):

\[ \Delta p = \frac{\Delta U}{c} = \frac{uA\Delta t}{c} = uA\Delta t \]

\[ \Delta p = 2 \frac{\Delta U}{c} = 2 \frac{uA\Delta t}{c} = 2uA\Delta t \]

辐射压力公式：

\[ P = \frac{F}{A} = \frac{\Delta p}{\Delta t} = u = \frac{S}{c} \]

100% 吸收

\[ \Delta p = p \rightarrow P = \frac{S}{c} \]

100% 反射

\[ \Delta p = 2p \rightarrow P = \frac{2S}{c} \]

例子：太阳能量

- 太阳辐射到地球的平均强度为 \( S \approx 10^3 \text{ W/m}^2 \)

- 对于100% 吸收，平均辐射压力为：

\[ P = \frac{S}{c} = \frac{10^3 \text{ W/m}^2}{3 \times 10^8 \text{ m/s}} = 3.3 \times 10^{-6} \text{ N/m}^2 \]

- 对于100% 反射，平均辐射压力为：

\[ F = PA = 3.3 \cdot 10^{-6} \text{ N/m}^2 \cdot 1\text{ m}^2 = 3.3 \cdot 10^{-6} \text{ N} \]

天线

- 天线是导体的排列，用于传输和接收无线电波。

- 参数：增益、阻抗、频率、方向、极化等。

- 半波天线

- 共振天线

- 勒吉天线

- 螺旋天线

- 环形天线

- 多频段天线

- 折线天线

- 线性天线