Physics 202, Lecture 22

Today’s Topics

- Standing waves, driven oscillation
- Electromagnetic Waves (EM Waves)
  - The Hertz Experiment
  - Review of the Laws of Electro-Magnetism
  - Maxwell’s equation
  - Propagation of $\mathbf{E}$ and $\mathbf{B}$
  - The Linear Wave Equation
Standing Waves (cont)

- Standing waves with a string of given length $L$ are produced by waves of natural frequencies or resonant frequencies:

$$\lambda = \frac{2L}{n}; \; n = 1, 2, 3\ldots$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L} = \sqrt{\frac{T}{\mu}} \frac{n}{2L}$$
Forced (driven) Oscillation

- If in addition there is a driving force with its own frequency $\omega$: $F_0 \cos(\omega t)$, the equation becomes:

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} + F_0 \cos(\omega t)$$

- This equation can be solved analytically.
  At large $t$, the solution is:

$$x = A \cos(\omega t + \phi)$$

- With

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b}{2m}\right)^2}}$$

- At large $t$, the frequency is determined by driving $\omega$
- When $\omega=\omega_0$, amplitude is maximum $\rightarrow$ resonance
Resonance Amplitude

\[ A = \frac{F_0}{m} \sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b}{2m}\right)^2} \]
Demo: Hertz Experiment

In 1887, Heinrich Hertz first demonstrated that EM fields can transmit over space.
Review: Gauss’s Law / Coulomb’s Law

The relation between the electric flux through a closed surface and the net charge $q$ enclosed within that surface is given by the Gauss’s Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
Gauss’s Law for Magnetism

- The Gauss’s Law for the electric flux is a reflection of the existence of electric charge. In nature we have not found the equivalent, a magnetic charge, or monopole.

- We can express this result differently: if any closed surface as many lines enter the enclosed volume as they leave it.

\[ \oint \vec{B} \cdot d\vec{A} = 0 \]
Review: Faraday’s Law

- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit” or closed path.

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- Since

\[ \mathcal{E} = \oint \vec{E} \cdot d\vec{l} \]

- Then

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]
Review: Ampere’s Law

- A magnetic field is produced by an electric current is given by the Ampere’s Law

\[ \oint B \cdot d\ell = \mu_0 I \]

- A changing electric field will also produce a magnetic field

Finally;

\[ \oint B \cdot d\ell = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]
Maxwell Equations

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \quad \rightarrow \text{Gauss’s Law/ Coulomb’s Law} \]

\[ \oint \vec{B} \cdot d\vec{A} = 0 \quad \rightarrow \text{Gauss’s Law of Magnetism, no magnetic charge} \]

\[ \oint \vec{E} \cdot d\ell = -\frac{d\Phi_B}{dt} \quad \rightarrow \text{Faraday’s Law} \]

\[ \oint \vec{B} \cdot d\ell = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \rightarrow \text{Ampere Maxwell Law} \]

Also, Lotentz force Law \( \rightarrow \)

\[ \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \]

These are the foundations of the electromagnetism
EM Fields in Space

- Maxwell equations when there is no charge and current:

\[ \oint \mathbf{E} \cdot d\mathbf{A} = 0 \]
\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \]

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]

Differential forms:
(singel polarization)

\[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \]
\[ \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \]

\[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \]
\[ \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]
Linear Wave Equation

- Linear wave equation
  \[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]
  - Wave speed
  - Certain physical quantity

- Sinusoidal wave
  \[ y = A \sin\left(\frac{2\pi}{\lambda} x - 2\pi ft + \phi\right) \]
  - A: Amplitude
  - \( v = \lambda f \)
  - \( f \): Frequency
  - \( k = \frac{2\pi}{\lambda} \)
  - \( \omega = 2\pi f \)
  - \( \phi \): Phase
  - \( \lambda \): Wavelength

General wave: superposition of sinusoidal waves
Electromagnetic Waves

- **EM wave equations:**
  \[
  \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}, \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
  \]

- **Plane wave solutions:**
  \[
  E = E_{\text{max}} \cos(kx-\omega t+\phi) \quad B = B_{\text{max}} \cos(kx-\omega t+\phi)
  \]

- **Properties:**
  - No medium is necessary.
  - E and B are normal to each other
  - E and B are in phase
  - Direction of wave is normal to both E and B (EM waves are transverse waves)
  - Speed of EM wave:
    \[
    c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s}
    \]
  - \(E/B = E_{\text{max}}/B_{\text{max}} = c\)
  - Transverse wave: two polarizations possible
The EM Wave

Two polarizations possible (showing one)