Physics 202, Lecture 20

Today’s Topics

- AC Circuits with AC Source
- Resistors, Capacitors and Inductors in AC Circuit
  - RLC Series In AC Circuit
- Impedance
- Resonances In Series RLC Circuit
AC Circuit

- Find out current $i$ and voltage difference $\Delta V_R$, $\Delta V_L$, $\Delta V_C$.

Notes:
- Kirchhoff’s rules still apply!
- A technique called phasor analysis is convenient.
Resistors in an AC Circuit

1. $\Delta V - IR = 0$ at any time

$$i_R = \Delta V/R = I_{\text{max}} \sin \omega t, \quad I_{\text{max}} = \Delta V_{\text{max}}/R$$

The current through a resistor is in phase with the voltage across it.
Inductors in an AC Circuit

- $\Delta V - L\frac{di}{dt} = 0$

$\Rightarrow i_L = I_{\text{max}} \sin(\omega t - \pi/2)$

$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L}$,

$X_L = \omega L \rightarrow$ inductive reactance

$\Rightarrow$ The current through an inductor is $90^\circ$ behind the voltage across it.
Capacitors in an AC Circuit

- $\Delta V - \frac{q}{C} = 0$, $\frac{dq}{dt} = i$

- $i_L = I_{\text{max}} \sin (\omega t + \frac{\pi}{2})$

- $I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_C}$

- $X_C = \frac{1}{\omega C}$ → capacitive reactance

- The current through a capacitor is 90° ahead of the voltage across it.
Summary of Phasor Relationship

I_R and ΔV_R in phase

I_L 90° behind ΔV_L
ΔV_L 90° ahead of I_L

I_C 90° ahead of ΔV_C
ΔV_C 90° behind I_C

Note that we set ΔV w.r.t. I
The current at all point in a series circuit has the same amplitude and phase (set it be $i=i_{\text{max}} \sin(\omega t)$).

Thus:

- $\Delta v_R = I_{\text{max}} R \sin(\omega t + 0)$
- $\Delta v_L = I_{\text{max}} X_L \sin(\omega t + \pi/2)$
- $\Delta v_C = I_{\text{max}} X_C \sin(\omega t - \pi/2)$

**Voltage across RLC:**

$\Delta v_{R\text{LC}} = \Delta v_R + \Delta v_L + \Delta v_R$

$= I_{\text{max}} R \sin(\omega t)$

$+ I_{\text{max}} X_L \sin(\omega t + \pi/2)$

$+ I_{\text{max}} X_C \sin(\omega t - \pi/2) = \Delta V_{\text{max}} \sin(\omega t + \phi)$

**how to get them?**
The phasor of $\Delta v_{RLC}$ = vector sum of phasors for $\Delta v_R$, $\Delta v_L$, $\Delta v_C$.

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2}$$

$$= I_{max} \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Note: $X_L = \omega L$, $X_C = 1/(\omega C)$

$$\Delta V = \Delta V_{max} \sin(\omega t + \phi)$$
Current And Voltages in a Series RLC Circuit

\[ \Delta v_R = (\Delta V_R)_{\text{max}} \sin(\omega t) \]
\[ \Delta v_L = (\Delta V_L)_{\text{max}} \sin(\omega t + \pi/2) \]
\[ \Delta v_C = (\Delta V_C)_{\text{max}} \sin(\omega t - \pi/2) \]

\[ \Delta V_{\text{max}} = I_{\text{max}} \sqrt{R^2 + (X_L - X_C)^2} \]
\[ \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \]

\[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \]

Quiz: Can the voltage amplitudes across each components, \((\Delta V_R)_{\text{max}}\), \((\Delta V_L)_{\text{max}}\), \((\Delta V_C)_{\text{max}}\) larger than the overall voltage amplitude \(\Delta V_{\text{max}}\)?
Problem 4 (5 points)

Given this LRC circuit in series:

\[ R = 100 \ \Omega \]
\[ L = 25 \ \text{mH} \]
\[ C = 10 \ \mu\text{F} \]
\[ V = V_0 \sin \omega t \]
\[ V = 10 \ \text{V} \]
\[ \omega/2\pi = 500 \ \text{Hz} \]

a) Which of the following phasor diagrams qualitatively represents this circuit?
Impedance

- For general circuit configuration:
  \[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) , \quad \Delta V_{\text{max}} = I_{\text{max}} Z \]
  \( Z \): is called Impedance.

- e.g. RLC circuit: 
  \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

- In general impedance is a complex number. \( Z = Z e^{i\phi} \).
  It can be shown that impedance in series and parallel circuits follows the same rule as resistors.
  \( Z = Z_1 + Z_2 + Z_3 + \ldots \) (in series)
  \( 1/Z = 1/Z_1 + 1/Z_2 + 1/Z_3 + \ldots \) (in parallel)

(All impedances here are complex numbers)
Resonances In Series RLC Circuit

- The impedance of an AC circuit is a function of $\omega$.
  - e.g Series RLC:
    $$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$
  - when $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ (i.e. $X_L = X_C$)
    \[\rightarrow\] lowest impedance \[\rightarrow\] largest current \[\Rightarrow\] resonance

- For a general AC circuit, at resonance:
  - Impedance is at lowest
  - Phase angle is zero. (In phase)
  - $I_{\text{max}}$ is at highest
  - Power consumption is at highest
Summary of Impedances and Phases

<table>
<thead>
<tr>
<th>Circuit Elements</th>
<th>Impedance $Z$</th>
<th>Phase Angle $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$R$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$C$</td>
<td>$X_C$</td>
<td>$-90^\circ$</td>
</tr>
<tr>
<td>$L$</td>
<td>$X_L$</td>
<td>$+90^\circ$</td>
</tr>
<tr>
<td>$RC$</td>
<td>$\sqrt{R^2 + X_C^2}$</td>
<td>Negative, between $-90^\circ$ and $0^\circ$</td>
</tr>
<tr>
<td>$RL$</td>
<td>$\sqrt{R^2 + X_L^2}$</td>
<td>Positive, between $0^\circ$ and $90^\circ$</td>
</tr>
</tbody>
</table>
| $LC$             | $\sqrt{R^2 + (X_L - X_C)^2}$ | Negative if $X_C > X_L$  
                             Positive if $X_C < X_L$ |

*In each case, an AC voltage (not shown) is applied across the elements.*

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