Physics 202, Lecture 14

Today’s Topics

- Sources of the Magnetic Field (Ch. 30)
- Review of Biot-Savart Law
- Ampere’s Law
- Magnetism in Matter
Magnetic Fields (Biot-Savart): Summary

Current loop, distance x on loop axis (radius R):

\[ B_x = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}} \]

\[ B_{\text{center}} = \frac{\mu_0 I}{2R} \]

Center of arc (radius R, angle \( \theta \)):

\[ B_{\text{center}} = \frac{\mu_0 I\theta}{4\pi R} \]

Straight wire: finite length

\[ B = \frac{\mu_0 I}{4\pi a} (\cos\theta_1 - \cos\theta_2) \]

infinite wire:

\[ B = \frac{\mu_0 I}{2\pi a} \]
Ampere’s Law

Ampere’s Law:

\[ \oint B \cdot dl = \mu_0 I_{encl} \]

- applies to any closed path, any static B field
- useful for practical purposes only for situations with high symmetry
Ampere’s Law: B-field of ∞ Straight Wire

Use symmetry:
Choose loop to be circle of radius $R$ centered on the wire in a plane $\perp$ to wire.

Why?
Magnitude of $B$ is constant (function of $R$ only)
Direction of $B$ is parallel to the path.

\[
\oint \vec{B} \cdot d\vec{l} = \oint Br \, d\theta = 2\pi rB = \mu_0 I_{encl} = \mu_0 I
\]

\[
B = \frac{\mu_0 I}{2\pi r}
\]
B Field Inside a Long Wire

Total current $I$ flows through wire of radius $a$ into the screen as shown.

What is the B field inside the wire?

By symmetry -- take the path to be a circle of radius $r$:

$$\oint \vec{B} \cdot d\vec{l} = B 2\pi r$$

- Current passing through circle:

$$\oint \vec{B} \cdot d\vec{l} = B 2\pi r = \mu_0 I_{encl} \Rightarrow B_{in} = \frac{\mu_0 I r}{2\pi a^2}$$

$$I_{encl} = \frac{r^2}{a^2} I$$
B Field of a Long Wire

Inside the wire: \( (r < a) \)

\[
B = \frac{\mu_0 I}{2\pi} \frac{r}{a^2}
\]

Outside the wire: \( (r > a) \)

\[
B = \frac{\mu_0 I}{2\pi r}
\]
Ampere’s Law: Toroid

Toroid: \( N \) turns with current \( I \).

\( B_\theta = 0 \) outside toroid!

(Consider integrating \( B \) on circle outside toroid: net current zero)

\( B_\theta \) inside: consider circle of radius \( r \), centered at the center of the toroid.

\[
\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_o I_{encl} = \mu_o NI
\]

\[
B = \frac{\mu_0 NI}{2\pi r}
\]
B Field of a Solenoid

Inside a solenoid: source of uniform B field

Solenoid: current $I$ flows through a wire wrapped $n$ turns per unit length on a cylinder of radius $a$ and length $L$.

If $a << L$, the $B$ field is approximately contained within the solenoid, in the axial direction, and of constant magnitude.

In this limit, can calculate the field using Ampere's Law!
Ampere’s Law: Solenoid

The B field inside an ideal solenoid is:

\[ B = \mu_0 nI \]

\[ n = \frac{N}{L} \]

Ideal solenoid

| Segment 3 | at \( \infty \) |
Magnetism in Matter

The B field produced along the axis of a circular loop (radius R) by a current I is:

$$\vec{B} \approx \frac{\mu_0 \mu}{2\pi z^3} \hat{z}$$

typical dipole behaviour

$$\mu \text{ is the magnetic moment } = I \cdot \text{area}$$

and $z \gg R$

Materials are composed of particles that have magnetic moments -- *(negatively charged electrons circling around the positively charged nucleus).*

orbital angular momentum

spin angular momentum

$\mu_B = \frac{q_e \hbar}{2m_e} = 9.27 \times 10^{-24} \frac{J}{T}$

(quantum mechanics)  

Bohr magneton
Magnetization

Apply external B field $B_0$. Field is changed within materials by these magnetic moments.

**Magnetization**: total magnetic moment per unit volume

$$\vec{M} \equiv \frac{\mu_{\text{total}}}{V}$$

The B field in the material is

$$\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$$

Define H (magnetic field strength):

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

(Magnetic susceptibility:

$$\vec{M} = \chi \vec{H}$$

$$\vec{B} = \mu \vec{H} = \mu_0 (1 + \chi) \vec{H}$$

“permeability” $\mu \left( = \kappa_m \mu_0 \right)$
Magnetic Materials

Materials are classified by magnetic susceptibilities:

- **Paramagnetic** (aluminum, tungsten, oxygen, ...)
  
  Atomic magnetic dipoles line up with the field, increasing it. Only small effects due to thermal randomization: \( \chi \sim +10^{-5} \)

- **Diamagnetic** (gold, copper, water, ... as well as superconductors)
  
  - Applied field induces an opposing field; usually very weak \( \chi \sim -10^{-5} \)

- **Ferromagnetic** (iron, cobalt, nickel, ...)
  
  - Dipoles prefer to line up with the applied field (similar to paramagnetic), but tend to all line up the same way due to collective effects: very strong enhancements \( \chi \sim +10^{+3} - 10^{+5} \)

Magnetic susceptibility temperature dependent (above range of typical values at T=20 C)
Ferromagnets

- Dipoles tend to strongly align over small patches – “domains” (even w/o external magnetic field). With external field, the domains align to produce a large net magnetization.

“Soft” ferromagnets
- Domains re-randomize when magnetic field is removed

“Hard” ferromagnets
- Domains persist even when the field is removed
- “Permanent” magnets
- Domains may be aligned in a different direction in a new external field
- Domains may be re-randomized by sudden physical shock
- If temperature is raised above “Curie point” (770 °C for iron), domains will also randomize (like a paramagnet)