Driven/Damped Oscillation

Damping: Suppose viscous drag $F_d = -b \dot{x}$ acts on mass on spring.

\[ ma = m \ddot{x} = F = -kx - b \dot{x} \]

\[ m \dddot{x} + k \dot{x} + b \ddot{x} = 0 \]

\[ \ddot{x} + \frac{b}{m} \dot{x} + \frac{k}{m} x = 0 \]

Driven mass on spring:

Solution: $x(t) = A e^{-\frac{bt}{2m}} \sin(\omega t + \phi_0)$

Amplitude decays over time.

Also, frequency is shifted $\omega^2 = \frac{k}{m} - (\frac{b}{2m})^2$

Decay time $T = \frac{2\pi}{b} = \text{time to decrease to } e^{-1}$, pendulum "enfolding time".

Underdamped $T > T_c$

\[ \frac{2\pi}{b} > \frac{2\pi}{\sqrt{m/k}} \]

Critically damped $T = T_c$

\[ \frac{b}{2m} = \frac{\omega}{2\pi} \]

Overdamped $T < T_c$

\[ \frac{b}{2m} > \frac{\omega}{2\pi} \]
Driven Oscillators

\[ F(t) = F_0 \sin \omega t \]

Motor drives mass at \( \omega \)

\[ F = F_0 \sin \omega t - kx = mx = m \ddot{x} \]

\[ m \ddot{x} + kx = F_0 \sin \omega t \]

\[ \ddot{x} + \frac{k}{m} x = \frac{F_0}{m} \sin \omega t \]

\[ \ddot{x} + \omega_0^2 x = \frac{F_0}{m} \sin \omega t \]

Solution:

\[ x(t) = \frac{F_0}{m(\omega_0^2 - \omega^2)} \sin \omega t \]

\[ A(\omega) = \frac{F_0}{m} \frac{1}{\omega_0^2 - \omega^2} \]

Resonance:

\[ A \to \infty \text{ as } \omega \to \omega_0 \]

Driven + Damped \( \to \) finite \( A \) at resonance

Wikipedia "Harmonic Oscillator" very good.
Tacoma Narrows Bridge

Wind force drives torsional oscillation

Bridge oscillates at natural frequency $\omega_0$.

Damping defeated by wind force which drives oscillation.

= ANY wind would drive oscillation

Strong wind exceeded max amplitude

Problem: weakly damped natural frequency