Note: an object rotates freely about its CM

\[ \text{TORQUE} = \text{rotating effect of a force} = \text{analog of force for rotation} \]

ex. Door (top view)

hinge $\vec{F}_h$

All four forces have same magnitude - which is most effective at turning the door?

axis $\vec{F}_i$ - furthest from axis of rotation AND 1 $\rightarrow$ door

Wrench on bolt

Turning stronger as $r$ increases as $F$ increases as $\phi \rightarrow 90^\circ$

$F$ torque $\tau \equiv rF \sin \phi$

(Nm, ft-lbs)

Torque has direction CW vs. CCW

Physics: direction = axis of CCW rotation

$\vec{\tau} \equiv \vec{r} \times \vec{F}$

(\(\vec{r}\) out of page, \(\vec{F}\) into page)
"Right hand rule 1": curl fingers along rotation \Rightarrow \text{thumb points in } + \text{ direction}

"Right hand rule 2": swing fingers from \( \uparrow \) to \( \downarrow \) \Rightarrow \text{thumb points in } - \text{ direction}

Note: "Center of Gravity" = gravity-weighted average position

\[ \text{center of gravity} \]

\[ \text{center of mass} \]

\[ \text{Earth} \]

\[ cg = \text{cm at surface of Earth} \]

\[ g = \text{constant} \]
Rotational Dynamics

Simple example: "force couple"

\[ F = ma = mR\alpha \] for each one

α same for both (counter-clockwise)

\[ \alpha = \frac{\tau}{I} = \frac{2F}{(2m)R} = \frac{2FR}{(2m)R^2} \]

\[ 2FR = I \alpha \]

Newton's 2nd Law

Look at Power input:

\[ P = F\omega = 2FR\omega = TW \]

Example: engine dynamometer: "dePuy brake"

\[ \text{friction belt} \quad (R \circ \omega) \quad \text{cogwheels} \quad \text{Temp}(\omega) = (F_1 - F_2)R \]

\[ \Rightarrow \text{Adjust } F_1 + F_2 \text{ to get } \alpha = 0 \]

(increase friction)
Dyno measures torque \( P = \tau \omega \)

"Brake horse power" (bhp)

refers to engine at crankshaft

(neglects transmission losses)

Wheel Race - analysis

\[ f_s \quad \text{torques:} \quad f_s \quad \text{CW} \]

\[ f_s \quad \text{forces:} \quad f_s \quad \frac{Mg \sin \theta}{R} \rightarrow \]

rolling constraint \( a = R \alpha \)

\[ \tau = I \alpha \Rightarrow f_s R = I \alpha = \frac{I a}{R} \]

force:

\[ Mg \sin \theta - f_s = Ma \]

\[ \Rightarrow Mg \sin \theta = f_s = \frac{I a}{R^2} \]

\[ Mg \sin \theta - \frac{I a}{R^2} = Ma \]

\[ \Rightarrow Mg \sin \theta = (M + \frac{I}{R^2}) a \]

\[ a = \frac{Mg \sin \theta}{(M + \frac{I}{R^2})} \]

\[ = \frac{g \sin \theta}{1 + \frac{I}{MR^2}} \]

-effective translational

hoop:

\[ I = MR^2 \Rightarrow a = \frac{g \sin \theta}{2} \]

disk:

\[ I = \frac{1}{2} MR^2 \Rightarrow a = \frac{g \sin \theta}{3/2} \]

sphere:

\[ I = \frac{2}{5} MR^2 \Rightarrow a = \frac{g \sin \theta}{7/5} \]
2011 Aprilia RSV4 Factory

Horsepower: 152.6 @ 12,600 RPM

hp, ft-lb curves always cross at 5252 RPM

peak power (torque x RPM)

Torque: 73.4 ft-lb @ 9700 RPM

peak torque