2-D collisions

general \( \Rightarrow \) (2) momentum equations
\[
\begin{align*}
P_{xi} &= P_{xf} \\
P_{yi} &= P_{yf}
\end{align*}
\]

if elastic \( \Rightarrow \) (1) energy equation
\[
K_i = K_f
\]

Center of Mass \( (= \) Center of Momentum \( ) \)

Math technique which isolates effect of external forces (zero in collision problems)

Before \( m_1 \rightarrow \begin{array} {c} \text{C}_m \end{array} \)

after \( m \rightarrow \begin{array} {c} \text{C}_m \end{array} \)

\[
\begin{align*}
P_i &= m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} \\
&= P_f = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}
\end{align*}
\]

Let \( \vec{P}_c = (m_1 + m_2) \vec{V}_{cm} = m_1 \vec{V}_{1c} + m_2 \vec{V}_{2c} \)

\[
\begin{align*}
(m_1 + m_2) &= \text{const} \Rightarrow \vec{V}_{cm} = \text{constant} \ (\text{by external forces})
\end{align*}
\]

Only change to \( \vec{V}_{cm} = \frac{\vec{P}}{m} = \text{velocity of center of mass} \)

\[
\vec{V}_{cm} = \frac{d\vec{r}_{cm}}{dt} \Rightarrow \vec{r}_{cm} = \frac{\vec{\xi}}{m} = \text{mass-weighted position}
\]
Rockets

No external force on rocket + propellant

\[ \sum F_{\text{sys}} = 0 \]

Rocket accelerated by reaction force of propellant = "thrust"

Propellant is ejected at \( V_p \) w.r.t. rocket

\[ \Rightarrow V_p = -V_p + V_r \text{ in original rest frame} \]

Propellant loses mass at same rate doesn't change after leaves rocket

\[ m_r = -m_p \]

\[ \Rightarrow \frac{dm_p}{dt} = m_r a_r + m_p a_r = -\frac{\Delta p_r}{dt} = -m_p (-V_p + V_r) \]

Drop "t", looking at rocket

\[ \text{Fuel "specific impulse"} = \frac{m a + m V_p}{\text{mass flown time}} \}

Rocket Equation

\[ \text{thrust} (N) \quad \parallel \quad \text{mass} < 0 \quad V_r > 0 \]
Rocket eqn solution

Rocket starts with mass \( m_0 \)
ends with mass \( m_f = m_0 - m_{\text{fuel}} \)

\[ a = -\frac{m}{m} V_p = \text{"specific thrust"} \]
when \( m = m_{\text{fuel}} \)

\[ \frac{dv}{dt} = -V_p \frac{dm}{m} \]

integrate \( \int_{v_i}^{v_f} dv = -V_p \int_{m_i}^{m_f} \frac{dm}{m} = -V_p \ln \frac{m_0}{m_f} = V_p \ln \frac{m_0}{m_f} \)

\[ \frac{m_0}{m_f} = \text{(w/o air resistance)} \]

Exit: Shuttle solid-fuel boosters (SRB)

Thrust = 14 MN each = \( m V_p \)

\( m_{\text{fuel}} = 590,000 \text{ kg} \)

\( \frac{1}{3} \) total liftoff mass

\[ \Rightarrow \frac{m_0}{m_f} = 1.0 \]

\( m_i = 240 \text{ s} \Rightarrow V_p = \frac{m}{m} V_p \left( \frac{m_i}{m} \right) = \frac{14 \text{ MN}}{590,000 \text{ kg}} \cdot 240 \text{ s} \]

Specific thrust \( \frac{14 \text{ MN}}{590,000 \text{ kg}} = 23.7 \text{ m/s}^2 \)

\( V_p = \frac{5,700 \text{ m/s}}{2.4} \text{ (Chemistry)} \)

= acceleration w/o payload

\[ V_f = V_p \ln \frac{m_0}{m_f} = 5,700 \text{ m/s} \ln \frac{1.0}{0.4} = 5,200 \text{ m/s} \]

Note: orbital speed = 8,000 m/s \( \Rightarrow \) need more engines

Also, \( \sqrt{2gh} = 2,500 \text{ m/s} \), fairly small \( V_f \) reduction
Challenge Problem for Math Whizzes

Show that for vertical launch w/ no air drag,
\[ y(t) = V_0 \left[ t + \left( \frac{m_0}{m_i} - t \right) \ln \left( 1 - \frac{m_i}{m_i t} \right) \right] - \frac{1}{2} gt^2 \]

Handout - online - Model Rockets
Image - online - Saturn V assembly
Apollo 17 Saturn V assembly - three liquid-fuel stages

S-IVB: 1.0 MN thrust, 120,000 kg, 470 s specific impulse, 165 s + 335 s burns

S-II: 4.4 MN thrust, 480,000 kg, 420 s specific impulse, 360 s burn

S-IC: five F-1 engines, 34 MN thrust, 2,300,000 kg, 260 s specific impulse, 150 s burn

Under Images on course site
Model Rockets (aka “paper rockets”)

Model rocket engines are usually made with a couple types of black powder. Most of it is burned as propellant, and then a separate part ignites after a delay to eject the parachute. Estes is the standard manufacturer of rocket engines, and they have a standard designation, like C6-7.

The C designates the total impulse, with the following values: A=2.5 Ns, B=5 Ns, C=10 Ns, D=20 Ns. Since they all have the same propellant speed, 800 m/s, this just corresponds to larger and larger mass, since the impulse is equal to propellant mass times propellant speed.

The next number, 6 in this case, designates the average thrust, 6 N, which is impulse divided by burn time. Model rockets don't have constant thrust, they have more at the beginning to help launch, and then flatten out with a constant value. Here's the thrust profile for a C6 engine:

![Thrust profile for a C6 Engine](image)

The final number, 7 in this case, designates the delay time in seconds between engine shutoff and parachute ejection. This is therefore the coasting time. If it’s too short, the parachute is ejected on the way up, which is no fun; if it’s too long, the parachute is ejected too late during descent, which is also no fun.

The mass of a C6-7 engine is 60.9 g, including 12.5 g propellant. You can fit a C6-7 into the popular Estes Alpha III 12-inch single-stage rocket, which is a good beginner rocket. The Alpha III mass is 34 g without the engine.

1. Calculate the final speed \( v_f \) that an Alpha III/C6-7 rocket would have in the absence of air drag and gravity.
2. Calculate the final speed \( v_f \) that it would have with gravity (but still neglecting air drag).
3. Calculate the final height \( h \) that it would have with gravity but neglecting air drag. (This problem involves some tricky integrals.)

Note that the rocket reaches a much lower height since air drag is a strong effect.
ENGINE CHART

- Delays have a tolerance of plus or minus 10% or 1 second, whichever is greater.
- All Estes engines come complete with igniters and patented igniter plugs (Pat. No. 5,410,966 and 5,509,354). The Estes Igniter Plug makes engine ignition extremely reliable.
- Do not fly a rocket/engine combination whose liftoff weight exceeds the recommended maximum liftoff weight.

<table>
<thead>
<tr>
<th>Prod. No.</th>
<th>Engine Type</th>
<th>Total Impulse</th>
<th>Time Delay</th>
<th>Max. Lift Wt.</th>
<th>Max. Thrust</th>
<th>Thrust Duration</th>
<th>Initial Weight</th>
<th>Propellant Weight</th>
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<td>N-sec Sec. Oz.</td>
<td>g Newtons Lbs.</td>
<td>Sec. Oz. g</td>
<td>Oz. g</td>
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UPPER STAGE ENGINES (PURPLE LABEL)

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<th>Prod. No.</th>
<th>Engine Type</th>
<th>Total Impulse</th>
<th>Time Delay</th>
<th>Max. Lift Wt.</th>
<th>Max. Thrust</th>
<th>Thrust Duration</th>
<th>Initial Weight</th>
<th>Propellant Weight</th>
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<td>1504</td>
<td>1/2A3-4T</td>
<td>1.25 4 1.0 28</td>
<td>8.3 1.9</td>
<td>0.3 0.21 6.0 0.06 1.75</td>
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<td>1599</td>
<td>A8-5</td>
<td>2.50 5 2.0 57</td>
<td>13.3 3.0</td>
<td>0.5 0.62 17.6 0.11 3.12</td>
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<td>1607</td>
<td>B6-6</td>
<td>5.00 6 2.5 71</td>
<td>12.1 2.7</td>
<td>0.8 0.78 22.1 0.22 6.24</td>
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BOOSTER STAGE ENGINES (RED LABEL)

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<th>Total Impulse</th>
<th>Time Delay</th>
<th>Max. Lift Wt.</th>
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<td>15.3 3.4</td>
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PLUGGED ENGINES - FOR USE WITH ROCKET POWERED RACERS & R/C ROCKET GLIDERS (BLUE LABEL)

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<th>Prod. No.</th>
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<th>Max. Lift Wt.</th>
<th>Max. Thrust</th>
<th>Thrust Duration</th>
<th>Initial Weight</th>
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<td>A10-PT</td>
<td>2.50 None 3.0 85</td>
<td>13.0 2.9</td>
<td>0.8 0.26 7.4 0.13 3.78</td>
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<td>27.6 6.2</td>
<td>1.8 1.55 44.0 0.88 24.93</td>
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</table>

The data listed above is from randomly chosen production samples.
NOTE: The "*" designates a mini engine.

* There are 4 mini engines per package. All other engines are 3 per package.