General relation between conservative $F$ and $V$:

$$W_{\text{int}} = \int F \cdot \text{d}x = -\Delta U$$

(If force does work, not energy decreases)

by force, not against force \[\downarrow\] vector calc

$$\vec{F} = -\vec{\nabla} V \quad (\vec{\nabla} = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z})$$

or in one dimension \[\vec{F} = -\frac{dU}{dx}\]

Potential

Energy

Diagram

Equilibrium:

$$U(x)$$

$$F(x)$$

$$x_0$$

$$U'(x_0) = 0$$

$$F(x_0) = 0$$

Stable equilibrium: $F$ restores to equilibrium position

Unstable equilibrium: $F$ pushes away from equilibrium position

Total Ex. Ball on Hill

Energy Diagram

$U(x) + E$ line
Spring Gun - how high will ball go as a function of spring compression?

\[ V_p = \frac{1}{2} kx^2 \]

\[ V_g = mgy \]

\[ \Rightarrow mgy = \frac{1}{2} kx^2 \Rightarrow y = \frac{k}{2gm} x^2 \quad \text{quadruple compression} \]

(8.72)

Loop the Loop - how high must ball start to completely loop the loop?

\[ v = \sqrt{gh - 4gR} \quad \text{to stay on track} \]

\[ \Rightarrow \quad 2h > 5R \]

\[ h > \frac{5R}{2} \quad \text{ball needs to start \frac{1}{2}R above top of loop} \]
Power = rate of doing work
\[ dW = \vec{F} \cdot d\vec{r} \]
\[ \Rightarrow P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v} \]
Units: \( \frac{Nm}{s} = \frac{J}{s} = \text{Watt (W)} \)

\begin{align*}
\text{Note: } & 1 \text{ hp} = 746 \text{ W} = 0.75 \text{ kW} \\
& \text{US, Britain} \quad \text{EU, Europe}
\end{align*}

Calculate power any way you can keep track of rate of doing work

ex. Chairlift
\[ h = 300 \text{ m} \]
Chairs come every 5 seconds
\[ \Rightarrow \text{rate} = 1 \text{ chair/5 s} \]
\[ \Rightarrow \text{pair in gets off per \( \Delta t \)} \]
\[ \text{pair in gets on at } y = 0 \]
1 chair = 2 people = 200 kg with gear and chair
\[ \Rightarrow P = \frac{mgh}{\Delta t} = \frac{200 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 300 \text{ m}}{5 \text{ s}} = 120 \text{ kW} = 160 \text{ hp} \]

ex. Top Speed of Car
\[ P_{\text{drag}} = F_{\text{drag}} \cdot v = P_{\text{wheels}} \approx 0.70 \text{ Peng (transmission)} \]
\[ \Rightarrow \frac{1}{2} \rho AC_d v^2 v = 0.70 \text{ Peng} \]
\[ \Rightarrow v = \left( \frac{2 \times 0.7 \text{ Peng}}{\rho AC_d} \right)^{1/3} \]

"Handy Dandy Formula" \( v = \frac{125 \text{ km/h}}{\text{Peng}^{1/3}} \) brutal!

Susan 2.5i Outback 150 with v (175 hp)
example: Power to climb hill (against gravity)

Let \( v = 60 \text{ mi/h} = 100 \text{ km/h} = 30 \text{ m/s} = \frac{ds}{dt} \)

\[
\frac{dh}{dt} = \frac{d}{dt} (0.10s) = 0.10 \frac{ds}{dt} = 0.10v \]

\[
P_g = mg \frac{dh}{dt} = mg(0.10v) \]

\( m = 1000 \text{ kg} \) \( \Rightarrow \) \( P_g = 30 \text{ kW} = 40 \text{ hp} \) fraction of total power

Redo Skidmark Forensics with work/energy

\[
E = \frac{1}{2}mv^2
\]

\[
W_{ext} = \Delta E = -\frac{1}{2}mv^2
\]

\[
W_{ext} = fd = -\mu_kmgd \Rightarrow v^2 = 2\mu_kgd
\]