Work on free particle: 
\[ dW = F \cdot dr \]
\[ = F_{\parallel} ds \]
\[ v = \frac{ds}{dt} \]
\[ F_{\parallel} = \frac{m \cdot dv}{dt} = \frac{mdv}{ds} \frac{ds}{dt} \]
( Note: \( F_{\parallel} \) changes direction, not speed!)

Add Newton's Law: 
\[ F_{\parallel} = ma_{\parallel} = m \frac{dv}{ds} = \frac{dv}{ds} \frac{ds}{dt} \]

\[ F_{\parallel} = \frac{m}{s} \frac{dv}{ds} \]

\[ \Rightarrow dW = F_{\parallel} ds = m \frac{dv}{ds} dv \]

\[ W_{12} = \int_{v_1}^{v_2} F_{\parallel} dv = \int_{v_1}^{v_2} m \frac{dv}{ds} dv = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \]

\[ v_1 = K_2 - K_1 \]

Effect of \( F_{\parallel} \) from \( v_1 \) to \( v_2 \) is to change

\[ \frac{1}{2} m v_2^2 = K = \text{Kinetic Energy} \]

\[ \Rightarrow W = \Delta K \text{ for free particle} \]

Look back at kinematics

\[ v_2^2 - v_1^2 = 2a \Delta x \]

\[ \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = ma \Delta x = F \Delta x = W_{12} ! \]

Work/Energy combines kinematics with Newton's 2nd Law.
Classic constant force: near-earth gravity

Apply force $F$ to lift object to height $y$; $dW = F \, dy$

$F$ balances gravity $\Rightarrow dW = mg \, dy$

\[ W = \int mg \, dy = mg \, y \, ( + \text{const}) \]

Have to do work $mg \, y$ to raise object of mass $m$ a distance $y$.

But object not moving at end, where is work energy?

\[ V(y) = mg \, y \, ( + \text{const}) \]

= gravity's "potential" to do work

\[ \text{arbitrary where you set } y = 0 \]

since constant energy offset has no effect on dynamics.

(often helpful to choose $y = 0$ position to eliminate a $V(y)$ term.)
Classic Varying Force: Spring

\[ F_s = -k \Delta x \quad \text{Hooke's Law} \]

relaxed \quad stretched \quad \Delta x \quad \text{position} \quad \frac{k}{N/m} \quad \text{spring constant}

Apply force to stretch spring: spring stretches while applied force balances

\[ F_s = -F \quad \text{Spring force} \]

Apply force: \( F + F_s = 0 \)

\[ dW = F_s \, dx = -F_s \, dx = k \Delta x \, dx \]

\[ W = \int_0^{x_f} \frac{1}{2} k x^2 \, dx = \frac{1}{3} k x_f^3 \]

Have to do work \( \frac{1}{2} k x^2 \) to stretch/compress spring by \( \pm x \) from relaxed position.

But object isn't moving at \( x_f \Rightarrow k = 0 \)

Where did work energy go?

Potential Energy \( \frac{1}{2} k x^2 = U(x) \)

Springs' "potential" to do work (if object is released)
Why is spring force so important in physics? (It's not like we literally use springs a lot!)

Look at any force that depends on position.

\( x_0 \) is equilibrium position where \( F = 0 \).

Look at motion near equilibrium position:

\[
F(x) = F(x_0) + (x-x_0)F'(x_0) + \frac{(x-x_0)^2}{2}F''(x_0) + \ldots
\]

Any force looks like a spring close to equilibrium where \( F = 0 \)!!!

- Often model small departures from equilibrium as a spring = linear restoring force
  - sound waves in various media
  - pendulum (later in 201)
  - water waves
  - crystal oscillation (like quartz)
  - plasma waves
  - lots and lots more!

"Nonlinear Dynamics" studies how things differ when force is not a spring force = modern area of research!
Work - Kinetic Energy - Potential Energy

Back to our four examples:

1. Free particle \( W = \frac{1}{2}mv^2 = \Delta K \) particle increases its kinetic energy.

2. Friction \( W = \text{lost to heat} = Q \) heat transfer (thermo).

3. Spring \( W = \frac{1}{2}kx^2 = \Delta U \) work done on spring compresses it, increases potential energy.

4. Gravity \( W = mgy = \Delta U \) work done on object increases its potential energy.

In general:

\[ W = \Delta K + \Delta U + Q \]

Special case: Isolated system with no external work input and no friction.

\[ \Rightarrow W = \Delta K + \Delta U = 0 \Rightarrow K + U = \text{constant} \]

Used a lot.

Conservative force if:

1. Work is path independent \( W_1 = W_2 = W_3 \)

2. Work is zero around closed path

ex. gravity, spring

\[ W = 0 \]

Non-conservative if not above

ex. friction, air drag

\[ \Rightarrow \] more work done on longer path