Example Problem 2.58

(a) Catapult launches rocket from well so initial speed is 80.0 m/s, \( v_0 = 80.0 \text{ m/s}\).

(1) Engines accelerate rocket at \( a = 4.00 \text{ m/s}^2 \) up to height 1000 m.

(2) Rocket falls freely from 1000 m altitude.

(b) How long is rocket above ground?

Two phases: (1) engine burn phase

\[ v_0, h, \text{ a known (can ignore phase (a))} \]

(2) free fall phase, starting with positive speed

(1) time not known, relate velocity to distance

\[ v_f^2 - v_i^2 = 2a(y_f - y_i) = 2ah \]

\[ v_f^2 = v_i^2 + 2ah = 14,400 \text{ m}^2/\text{s}^2 \]

\[ v_f = 120 \text{ m/s at engine cutoff} \quad \Rightarrow \quad t_1 \] from acceleration

\[ v_f - v_0 = at \quad \Rightarrow \quad t_1 = \frac{v_f - v_0}{a} = \frac{40.0 \text{ m/s}}{4.00 \text{ m/s}^2} \]

\( \Rightarrow \quad t_1 = 10.0 \text{ s} \)

(2) free fall with initial height 1000 m, initial speed +120 m/s

\[ y = h + v_i t_i - \frac{1}{2} g t_i^2 \] solve for \( y = 0 \) (ground)
(2) 2.58 cont.

\[ 0 = h + v_1 t_2 - \frac{1}{2} g t_2^2 \]

requires quadratic eqn

\[ t_2 = \frac{-v_1 \pm \sqrt{v_1^2 + \frac{4gh}{2g}}}{-\frac{1}{2}g} \]

\[ = \frac{+v_1 + \sqrt{v_1^2 + 2gh}}{-\frac{1}{2}g} \]

\[ t_2 \approx 0.70 \text{ s} \]

\[ t_2 = \frac{120 \text{ m/s} + \sqrt{(120 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(1000 \text{ m})}}{9.8 \text{ m/s}^2} = 31.1 \text{ s} \]

\[ \text{total time} = t_1 + t_2 = 41.1 \text{ s} \]

(b) Highest altitude? Again it doesn’t matter, want \( y_{\text{max}} \) when \( v = 0 \)

\[ v^2 = u^2 - 2g(y - h) = 0 \]

\[ u = 120 \text{ m/s} \]

\[ 9 = \frac{v^2}{2g} + h = 1.735 \text{ m} \]

(c) Speed at ground? Falls from rest at 1735 m

\[ v_{\text{gr}}^2 = 2gh_{\text{max}} \Rightarrow v_{\text{gr}} = \sqrt{2gh_{\text{max}}} = 184 \text{ m/s} \]
CH 3 - Vectors

Cartesian Grid - Foundation of description of objects in space: \((x, y, z)\)

20\textsuperscript{th} Century: add time as a coordinate

\((x, y, z, t) = \text{"space-time" location} = \text{4-vector}\)

We'll stick with 3-D space vectors

\underline{Position Vector} = \text{arrow from origin (0,0) to position (x, y)}

\[ \| \vec{r} \| = r = \text{distance from origin} \]

\(\vec{r}\) includes direction

\underline{Examples}:

Scalars \(?\) vs. Vectors

- time
- temperature
- power
- speed
- position
- displacement
- force
- velocity

\underline{Displacement = \text{change of position}}

"Walk two blocks east" \(\rightarrow 2 \text{ blocks}\)

Can be found via vector subtraction
Adding vectors: tail to tip

Subtracting vectors

1) Reverse second (changes sign) and ADD:

2) Tail to Tail, arrow points to first head
Vectors on 2-D step-motion diagrams

\[ \Delta \vec{r} = \text{displacement} \]
\[ \Delta t = \text{time} \]

Unit Vectors - carry only direction
\[
\text{magnitude } = \frac{1}{1} \text{ (no units)}
\]

e.g., let \( \hat{i} \) = unit vector in \( x \) direction
or East

\[ \Rightarrow \text{"3 miles east"} = (3 \text{ mi}) \hat{i} \]

\[ \vec{r} = x \hat{i} + y \hat{j} + z \hat{k} \]
\[ \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \]
\[ \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \]

\[ \Rightarrow \vec{A} - \vec{B} = (A_x - B_x) \hat{i} + (A_y - B_y) \hat{j} + (A_z - B_z) \hat{k} \]