Preliminary Mathematics Problems

1. \[ax^2 + bx + c = 0\]
   Solve for \(x\) in terms of \(a, b, c\)

2. a) \(\ln(e^a) = \)
   b) \(\ln(xy) = \)
   c) \(\ln\left(\frac{x}{y}\right) = \)
   d) \(\ln(a^x) = \)
   e) \(e \approx \) (3 digits)

3. a) \(\log(10^n) = \)
   b) \(\log(1000) = \)
   c) \(\log(0.01) = \)

4. See figure at right:
   a) Area of circle =
   b) Circumference of circle =
   c) \(360^\circ = ? \) radians
   d) \(\pi \approx \) (3 digits)

5. See figure. Express in terms of \(a, b \& c\):
   a) \(\sin \alpha = \)
   b) \(\cos \alpha = \)
   c) \(\sin \beta = \)
   d) \(\cos \beta = \)
   e) \(\tan \alpha = \)

6. f) Express \(c\) in terms of \(a \& b\):
   g) Express \(\alpha\) in terms of \(\beta\):
   h) Area of triangle =
7. Simplify expressions
   a) \( \sin(-\alpha) = -\sin \alpha \) [Example]
   b) \( \cos(-\alpha) = \)
   c) \( \sin(90^\circ - \alpha) = \)
   d) \( \cos(90^\circ - \alpha) = \)
   e) \( \sin(180^\circ - \alpha) = \)
   f) \( \sin(180^\circ + \alpha) = \)

8. \( \sin 0^\circ = \)
   \( \cos 0^\circ = \)
   \( \sin 90^\circ = \)
   \( \cos 90^\circ = \)
   \( \sin 180^\circ = \)
   \( \cos 180^\circ = \)
   \( \sin 270^\circ = \)
   \( \cos 270^\circ = \)
   \( \tan 0^\circ = \)
   \( \tan 45^\circ = \)

9. See figure below:

   ![Triangle Diagram](image)

   a) Express \( C \) in terms of \( A, B \) & \( \gamma \).
   b) " \( B \) in terms of \( A, C \) & \( \beta \).
   c) " \( A \) in terms of \( B, C \) & \( \alpha \).

10. Same triangle above:
    a) Express \( A \) in terms of \( B, \beta \) & \( \alpha \)
    b) " \( B \) in terms of \( A, \beta \) & \( \alpha \)
        \( B \) in terms of \( C, \beta \) & \( \gamma \)
        \( C \) in terms of \( A, \alpha \) & \( \gamma \)

11. Express the area of the (same) triangle:
    a) in terms of \( A, C \) & \( \beta \)
    b) in terms of \( A, B \) & \( \gamma \).
    c) Express \( \alpha \) in terms of \( \beta \) & \( \gamma \).
12. a) Volume of a sphere of radius $r$
   b) Surface area of a sphere of radius $r$.

13. Metric prefixes:

   - $k$ means $10^3$, is written **kilo**. [example]
   - **p**
   - **n**
   - **μ**
   - **m**
   - **M**
   - **G**

14. Write down the first and second derivatives of the following functions
   (a, b, c & $\omega$ are constants)

   a) $\frac{d}{dx} (ax + b) =$
   f) $\frac{d^2}{dx^2} (ax + b) =$

   b) $\frac{d}{dx} (ax^2 + bx + c) =$
   g) $\frac{d^2}{dx^2} (ax^2 + bx + c) =$

   c) $\frac{d}{dx} \left( \frac{a}{x} \right) =$
   h) $\frac{d^2}{dx^2} \left( \frac{a}{x} \right) =$

   d) $\frac{d}{dt} \sin(\omega t + a) =$
   i) $\frac{d^2}{dt^2} \sin(\omega t + a) =$

   e) $\frac{d}{dt} \cos(\omega t + a) =$
   j) $\frac{d^2}{dt^2} \cos(\omega t + a) =$
15. Write down the indefinite and definite integrals of the following functions

a) \( \int a \, dx = \)

b) \( \int (ax + b) \, dx = \)

c) \( \int (ax^2 + bx + c) \, dx = \)

d) \( \int \cos (wt) \, dt = \)

e) \( \int \sin (wt) \, dt = \)

f) \( \int_1^2 \alpha \, dx = \)

g) \( \int_0^2 (ax + b) \, dx = \)

h) \( \int_0^2 (ax^2 + bx + c) \, dx = \)

i) \( \int_0^{\pi/2w} \cos (wt) \, dt = \)

j) \( \int_0^{\pi/2w} \sin (wt) \, dt = \)

16. Given the function: \( x(t) = -3t^2 + 13t + 10, \) find the maximum value of \( x \) and the value of \( t \) for which this maximum occurs.