Physics 241  fall 2010
HW set #2

1-19

\[ u' = 0.9 \quad \quad u'' = 0.9 \]
\[ u' = \frac{u'' + u'/c}{1 + u''u'/c} = 1.8 \quad 0.994 \]
\[ u = \frac{u'' + u'/c}{1 + u''u'/c} = \frac{1.8945}{1.8946} = 0.99995 \]

1-22 \( d = 2.5 \times 10^3 \) light years

\[ \frac{c}{3600 \text{ sec}} = 1000 \times 10^3 \text{ m/s} = 278 \text{ m/s} \]

In the jet: \( x' = \gamma (x - \beta c x) \)
\[ \beta = 9.2 \times 10^{-7} \quad \gamma = 1 + \frac{1}{2} \beta^2 \]

In the jet: \( x' = \gamma (x - \beta c x) \)

\( \gamma > 1 \)
\( \gamma < 1 \)

Here: \( \Delta x' = c x'_0 - c x'_l = 2 \gamma \delta d \)

We can set \( \gamma = 1 \)
\( c = 2.5 \times 10^3 \) c years

\[ \Delta x' = 5 \times 10^3 \times 9.2 \times 10^{-7} \text{ years} = 4.6 \times 10^3 \text{ years} = 1.8 \text{ days} \]

Lyra comes in first 

1-23

\[ u' = 0.6 \quad \quad x' = 1.25 \quad \quad x = x(x' + \beta c x') \]
\[ c x' = \beta (c x' + \beta x') \]
1-23 Until to measure the length, you need to look at
the two ends at the same time in your frame (0)
\[ cT_1 = 0 = cT_1' \]
\[ cT_2 = 0 = cT_2' + \beta \]
\[ x_1 = 0 \]
\[ x_2 = x(-1 + \beta^2) = -\frac{x}{\beta} \]

Length: \( L \) in 0 = 0.8 m

The far end crosses the origin in 0 in time \( t = \frac{L}{c} \).

\[ \beta \approx 0.75 \]

or \( cT = \frac{L}{c} = 1.33 \text{ meters} \).

Omit part C

\[ t = 4.4 \times 10^{-9} \text{ sec.} \]

1-33 \( \beta = 10^{-3}, 10^{-2}, 10^{-1} \) Doppler shift's receding

\[ f = f_o \left( \frac{1 - \beta}{1 + \beta} \right)^{\frac{1}{2}} = \frac{f_o}{\lambda} \]

So \( \lambda = \lambda_o \left( \frac{1 + \beta}{1 - \beta} \right)^{\frac{1}{2}} \) upside down.

For small \( \beta \)

\[ \lambda \approx \lambda_o (1 + \beta) \]

\[ \beta \]

\[ \frac{1 + \beta}{\beta} \]

\[ \lambda \left[ \text{NW} \right] \]

\[ 0.001 \quad 1.001 \quad 686.95 \]

\[ 0.01 \quad 1.01 \quad 662.86 \]

\[ 0.1 \quad 1.05 \quad 725.21 \]

1-43 \( \Pi^+ \) mean beam, \( \beta = 0.92 \), \( \gamma = 2.55 \)

\[ T_\text{in} = 2.6 \times 10^{-8} \text{ sec.} \]

\[ T_\text{lab} = \gamma T_\text{in} = 6.63 \times 10^{-8} \text{ sec.} \]

6 cm = 20 meters. \( cT = 7.8 \text{ m} \)

After 50 meters, \( e^{-25} = 0.082 \times 50,000 = 4,100 \Pi^+ \)s remain

Without time dilation \( e^{-6.4} = 1.6 \times 10^{-3} \times 50,000 = 82 \Pi^+ \)s remain

1-53 The tilt appears because in 0, to measure the positions of the two ends of the stick you need equal times \( cT \), but those are different in 0'.

Note \( cT \), so one end of the stick may travel further than the other. \( cT' \) is length invariant.

\[ cT' = x(cT' - \beta x) \]

For \( x = 0 \) \( cT = cT'/\gamma \)

For \( x = l \) \( cT = cT'/\gamma + \beta L \) is not correct.

\[ \Delta x = \beta L \]

\( \Delta y = v_o \Delta t = \beta y_o \tan \theta \cdot \beta \frac{y_o}{c} = 0.85 \frac{y_o}{c} \)
\( \text{1-53 cont'd:} \) Length measurement

\[ x' = \gamma (x - \beta ct) \]

\[ ct' = \gamma (ct - \beta x) \]

\[ ct = \beta \sqrt{\frac{l^2}{\gamma} + (\beta y)^2} \]

\[ x' = \gamma (l - \beta^2 \sqrt{\frac{l^2}{\gamma}}) \]

So the length in \( O' \) of the tilted stick is

\[ l' = \left( \frac{\beta^2}{\gamma} + \frac{l^2 - \beta^2 \sqrt{\frac{l^2}{\gamma}}}{\gamma} \right)^{1/2} = l \left( 1 - \frac{\beta^2}{\gamma} \right)^{1/2} \]

\[ \Delta y = \frac{\Delta t}{\gamma \sqrt{1 - \beta^2 / \gamma}} \text{ unless the stick is really moving fast!} \]

Addendum to #1-23 (not required)

Part c can be correctly done by translating the length and working in the \( O' \), space-time frame.

\[ \beta = 0.6 = \tan \theta \]

\[ \gamma = 1.25 \]

\[ ct' = 0 \]

\[ ct_2' = \frac{l}{\gamma \sqrt{1 - \beta^2 / \gamma}} \]

So this type of figure works. But the length \( l' / \gamma \)

has to be calculated via the Lorentz transformation.