Physics 241
HW set #10 Chap 6-9, 14, 15, 24, 29, 36

9. Infinite well $0 \leq x \leq L$

$L = 0.1 \text{ nm}$

$E_n = \frac{n^2 \hbar^2}{2m}$

$k_n = n \pi \quad \psi_n(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right)$

$E_1 = \frac{\hbar^2 \pi^2}{2mL^2}$

$a $a$ for a proton $m_e = 9.31 \times 10^{-6}$

$E_1 = \frac{4 \times 10^2 \times 10^{-16}}{1.86 \times 10^{-2}} = 2.1 \times 10^{-2} \text{ eV}$

$L = 1 \times 10^{-15} \text{ m} = 1 \times 10^{-6} \text{ mm}$

$E_1 = \frac{4 \times 10^2 \times 10^{-12}}{1.86 \times 10^{-12}} = 2.1 \times 10^8 \text{ eV} = 210 \text{ MeV}$

Too high. The confinement of a proton in a nucleus is not as strong as an infinite well. Recall that an electron in $L = 0.1 \text{ nm}$ has $E_1 = 40 \text{ eV}$, which is also too high.

14. Tube of length $L$ (infinite well)

$\Delta x = L$  $\Delta p = \left(\frac{1}{\hbar}\right)$

$E = \frac{\Delta p^2}{2m} = \frac{\hbar^2}{2mL^2}$

not bad - off by $\hbar^2$

Infinite well $E_1 = \frac{\hbar^2 \pi^2}{2mL^2}$

factor of 10 larger.

We would have done better to use $k$ instead of $n$, but that's the uncertainty principle. It is not precise.

15. For the above problem $m = 1 \quad \lambda = 2L$

$m = 2 \quad \lambda = \frac{L}{2}$

$m = 3 \quad \lambda = \frac{L}{3}$

$p = \frac{\hbar}{\lambda} = \frac{2\pi \hbar}{2L}$

$E = \frac{p^2}{2m} = \frac{\pi^2 \hbar^2}{2mL^2}$

which is correct.
24. Finite square well, \( A = 1.0 \times 10^{-6} \) mm

For two energy levels,

\[
\frac{mV_0a^2}{2\hbar^2} = \frac{\pi^2}{4}
\]

\[
V_0 = \frac{\pi^2 \cdot 2\hbar^2 c^2}{4mc^2a^2} = \frac{10 \times 4 \times 10^4 \times 10^{-16}}{2 \times 938 \times 10^{-12}} = 2.1 \times 10^8 \text{ eV} = 2.1 \text{ MeV}
\]

\( a \) is about \( \frac{1}{3} \) as large as it should be for the deuteron, which has \( V_0 \approx 20 \text{ MeV} \).

29. \( \psi(x) = \left( \frac{2}{L} \right)^{1/2} \sin \frac{3\pi x}{L} \)

\[
\langle x \rangle = \frac{1}{L} \int_0^L x \sin \frac{3\pi x}{L} dx
\]

\[
\langle x^2 \rangle = \frac{1}{L} \int_0^L x^2 \sin \frac{3\pi x}{L} dx
\]

\[
= \frac{2}{L} \left( \frac{L^3}{6} - \frac{L^3}{18} \right) = \frac{L^2}{3} \left( 1 - \frac{1}{3n^2} \right) = 0.33L^2
\]

Note: \( \langle x \rangle \) was obtained by symmetry.

36. 10 grand state \( m = 0 \)

\( E_0 = \frac{\hbar^2}{2L^2} \)

\( H_0(\beta x) = \beta \)

\[
\psi_0(x) = \left( \frac{\beta}{\sqrt{\pi}} \right)^{1/2} e^{-\beta^2 x^2/2}
\]

\[
\beta^2 = \frac{m\omega}{\hbar}
\]

\[
\psi_0 = \frac{\beta}{\sqrt{\pi}} e^{-\beta^2 x^2} \int_{-\infty}^{+\infty} \psi_0^2(x) dx = \left( \frac{\beta}{\sqrt{\pi}} \right)^{1/2} \int_{-\infty}^{+\infty} e^{-\beta^2 x^2} dx = 1
\]

\[
\langle x^2 \rangle = \frac{\beta}{\sqrt{\pi}} \int_{-\infty}^{+\infty} x^2 e^{-\beta^2 x^2} dx = \frac{\beta^2}{2\beta^2} \int_{-\infty}^{+\infty} e^{-\beta^2 x^2} dx
\]
\[ \int_{-\infty}^{+\infty} e^{-\beta x^2} dx = \frac{\sqrt{\pi}}{\beta} = \frac{\sqrt{\pi}}{\sqrt{\beta^2}} \]

\[ \frac{2}{\beta^2} \sqrt{\pi} \left( \beta^2 \right)^{-\frac{1}{2}} - \frac{\sqrt{\pi}}{2} \left( \beta^2 \right)^{-1} = \frac{\sqrt{\pi}}{2} \frac{1}{\beta^3} \]

\[ \langle x^2 \rangle = \frac{\beta \sqrt{\pi}}{\beta^3} \frac{1}{2 \beta^2} = \frac{1}{2} \frac{1}{2 \beta^2} = \frac{1}{2m\omega_0} \]

\[ \langle V(x) \rangle = \frac{1}{2} k \langle x^2 \rangle = \frac{1}{4} \frac{k}{m\omega_0} = \frac{1}{4} m\omega_0 = \frac{E_0}{2} \]

\( \langle V(x) \rangle \) and \( \langle KE \rangle \) are equal, and each equal to \( \frac{E_0}{2} \)