Problems

Level I

Section 1-1 The Experimental Basis of Relativity

1-1. In episode 5 of Star Wars, the Empire’s spaceships launch probe droids throughout the galaxy to seek the base of the Rebel Alliance. Suppose a spaceship moving at $2.3 \times 10^4$ m/s toward Hoth (site of the rebel base) launches a probe droid toward Hoth at $2.1 \times 10^8$ m/s relative to the spaceship. According to Galilean relativity, (a) What is the speed of the droid relative to Hoth? (b) If rebel astronomers are watching the approaching spaceship through a telescope, will they see the probe before it lands on Hoth?

1-2. In one series of measurements of the speed of light, Michelson used a path length $L$ of 27.4 km (17 mi). (a) What is the time needed for light to make the round trip of distance $2L$? (b) What is the classical correction term in seconds in Equation 1-5, assuming Earth’s speed is $v = 10^{-4}c$? (c) From about 1600 measurements, Michelson arrived at a result for the speed of light of 299,796 ± 4 km/s. Is this experimental value accurate enough to be sensitive to the correction term in Equation 1-5?

1-3. A shift of one fringe in the Michelson-Morley experiment would result from a difference of one wavelength or a change of one period of vibration in the round-trip travel of the light when the interferometer is rotated by 90°. What speed would Michelson have computed for Earth’s motion through the ether had the experiment seen a shift of one fringe?

1-4. In the “old days” (circa 1935) pilots used to race small, relatively high-powered airplanes around courses marked by a pylon on the ground at each end of the course. Suppose two such evenly matched racers fly at airspeeds of 130 mph. (Remember, this was a long time ago!) Each flies one complete round trip of 25 miles, but their courses are perpendicular to each other and there is a 20-mph wind blowing steadily parallel to one course. (a) Which pilot wins the race and by how much? (b) Relative to the axes of their respective courses, what headings must the two pilots use?

1-5. Paul Ehrenfest suggested the following thought experiment to illustrate the dramatically different observations that might be expected, dependent on whether light moved relative to a stationary ether or according to Einstein’s second postulate:

Suppose that you are seated at the center of a huge dark sphere with a radius of $3 \times 10^4$ m and with its inner surface highly reflective. A source at the center emits a very brief flash of light that moves outward through the darkness with uniform intensity as an expanding spherical wave.

What would you see during the first 3 seconds after the emission of the flash if (a) the sphere moved through the ether at a constant 30 km/s and (b) if Einstein’s second postulate is correct?

1-6. Einstein reported that as a boy he wondered about the following puzzle. If you hold a mirror at arm’s length and look at your reflection, what will happen if you begin to run? In particular, suppose you run with speed $v = 0.99c$. Will you still be able to see yourself? If so, what would your image look like, and why?

1-7. Verify by calculation that the result of the Michelson-Morley experiment places an upper limit on Earth’s speed relative to the ether of about 5 km/s.

1-8. Consider two inertial reference frames. When an observer in each frame measures the following quantities, which measurements made by the two observers must yield the same results? Explain your reason for each answer.

(a) The distance between two events
(b) The value of the mass of a proton
(c) The speed of light
(d) The time interval between two events
(e) Newton’s first law
(f) The order of the elements in the periodic table
(g) The value of the electron charge

Section 1-2

1-9. Assume a clock at $C'$ and is measured $C$ is measured at $C$. Let $C'$ be moving at $v$ in the $+x$ direction and $C$ be stationary. A flash occurs at $C$ and light waves move from $C$ to both $C'$ and $P$. What is the difference in the time that the waves arrive at $P$?

1-10. Suppose a clock at $C'$ and is measured at $C$. Let $C'$ be moving in the $+x$ direction with a speed of $v$. A flash occurs at $C$ and light waves move from $C$ to both $C'$ and $P$. What is the difference in the time that the waves arrive at $P$?

1-11. Make up a thought experiment that demonstrates a paradox with the results of Einstein’s theory.

1-12. Two reference frames $S$ and $S'$ are at rest with respect to each other. (a) Use the Lorentz factor $\gamma$ to show that a clock at rest in $S$ reads less time than a clock moving with respect to $S$ with speed $v$. (b) Use the Lorentz factor $\gamma$ to show that a clock moving with respect to $S$ reads less time than a clock at rest in $S$.

1-13. Suppose a clock moves with respect to a stationary clock with a speed of $v = 0.4c$. The origins of the two clocks coincide at time $t_0$. How long does it take for the moving clock to complete one cycle if the stationary clock completes one cycle in 20 seconds?

1-14. Show that the duration of the oscillation of a mechanical pendulum is independent of the earth’s motion (but only if the pendulum is not near the earth’s surface).

1-15. Show that $c$ and $v$ are not related by any formula such as $c = v + v^2$.

1-16. Show that a clock at rest in $S$ reads less time than a clock moving with respect to $S$.

1-17. Consider a rocket moving with constant acceleration. How might the time interval measured by the rocket clock compare with the time interval measured by an observer in $S$?

1-18. A light ray strikes a mirror at right angles to the velocity of the light ray in $S$ is $c$. Find the velocity of the light ray in $S$.

1-19. A particle moves along the positive x direction with a speed of $0.9c$. Find the velocity of the particle in the rest frame of the particle.

1-20. Use the result of the problem above to determine what the particle looks like when observed by an observer in the rest frame of the particle.

1-21. How would you change the Lorentz transformations to account for motion in a gravitational field?
Section 1-2 Einstein's Postulates

1-9. Assume that the train shown in Figure 1-14 is 1.0 km long as measured by the observer at C' and is moving at 150 km/h. What time interval between the arrival of the wave fronts at C' is measured by the observer at C in S?

1-10. Suppose that A', B', and C' are at rest in frame S', which moves with respect to S at speed v in the +x direction. Let B' be located exactly midway between A' and C'. At t' = 0 a light flash occurs at B' and expands outward as a spherical wave. (a) According to an observer in S', do the wave fronts arrive at A' and C' simultaneously? (b) According to an observer in S, do the wave fronts arrive at A' and C' simultaneously? (c) If you answered no to either (a) or (b), what is the difference in their arrival times and at which point did the front arrive first?

Section 1-3 The Lorentz Transformation

1-11. Make a graph of the relativistic factor $\gamma = 1/(1 - v^2/c^2)^{1/2}$ as a function of $\beta = v/c$. Use at least 10 values of $\beta$ ranging from 0 up to 0.995.

1-12. Two events happen at the same point $x_0'$ in frame S' at times $t'_1$ and $t'_2$. (a) Use Equation 1-19 to show that in frame S, the time interval between the events is greater than $t'_3 - t'_1$ by a factor $\gamma$. (b) Why is Equation 1-18 less convenient than Equation 1-19 for this problem?

1-13. Suppose an event occurs in inertial frame S with coordinates $x = 75$ m, $y = 18$ m, $z = 4.0$ m at $t = 2.0 \times 10^{-5}$ s. The inertial frame S' moves in the +x direction with $v = 0.85c$. The origins of S and S' coincide at $t = t' = 0$. (a) What are the coordinates of the event in S'? (b) Use the inverse transformation on the results of (a) to obtain the original coordinates.

1-14. Show that the null effect of the Michelson-Morley experiment can be accounted for if the interferometer arm parallel to the motion is shortened by a factor of $(1 - v^2/c^2)^{1/2}$.

1-15. Two spaceships are approaching each other. (a) If the speed of each is 0.9c relative to Earth, what is the speed of one relative to the other? (b) If the speed of each relative to Earth is 30,000 m/s (about 100 times the speed of sound), what is the speed of one relative to the other?

1-16. Starting with the Lorentz transformation for the components of the velocity (Equation 1-23), derive the transformation for the components of the acceleration.

1-17. Consider a clock at rest at the origin of the laboratory frame. (a) Draw a spacetime diagram that illustrates that this clock ticks slow when observed from the reference frame of a rocket moving with respect to the laboratory at $v = 0.8c$. (b) When 10 s have elapsed on the rocket clock, how many have ticked by on the lab clock?

1-18. A light beam moves along the y' axis with speed c in frame S', which is moving to the right with speed v relative to frame S. (a) Find $u_x$ and $u_y$, the x and y components of the velocity of the light beam in frame S. (b) Show that the magnitude of the velocity of the light beam in S is c.

1-19. A particle moves with speed 0.9c along the x' axis of frame S', which moves with speed 0.9c in the positive x direction relative to frame S. Frame S' moves with speed 0.9c in the positive x direction relative to frame S. (a) Find the speed of the particle relative to frame S'. (b) Find the speed of the particle relative to frame S.

Section 1-4 Time Dilation and Length Contraction

1-20. Use the binomial expansion to derive the following results for values of $v \ll c$ and use when applicable in the problems that follow.

(a) \[ \gamma \approx 1 + \frac{1}{2} \frac{v^2}{c^2} \]

(b) \[ \frac{1}{\gamma} \approx 1 - \frac{1}{2} \frac{v^2}{c^2} \]

(c) \[ \gamma - 1 \approx 1 - \frac{1}{2} \frac{v^2}{c^2} \]

1-21. How great must the relative speed of two observers be for their time-interval measurements to differ by 1 percent (see Problem 1-20)?
1.22. A nova is the sudden, brief brightening of a star (see Chapter 13). Suppose Earth astronomers see two novas occur simultaneously, one in the constellation Orion (the Hunter) and the other in the constellation Lyra (the Lyre). Both novas are the same distance from Earth, \(2.5 \times 10^4 \, \text{pc} \cdot \text{y} \), and are in exactly opposite directions from Earth. Observers on board an aircraft flying at 1000 km/h on a line from Orion toward Lyra see the same novas, but note that they are not simultaneous. (a) For the observers on the aircraft, how much time separates the nova? (b) Which one occurs first? (Assume Earth is an inertial reference frame.)

1.23. A meter stick moves parallel to its length with speed \( v = 0.6c \) relative to you. (a) Compute the length of the stick measured by you. (b) How long does it take for the stick to pass you? (c) Draw a spacetime diagram from your viewpoint of the frame with the front of the meter stick at \( x = 0 \) when \( t = 0 \). Show how the answers to (a) and (b) are obtained from the diagram.

1.24. The proper mean lifetime of \( \pi \) mesons (pions) is \( 2.6 \times 10^{-8} \, \text{s} \). If a beam of such particles has speed 0.9c, (a) What would their mean life be as measured in the laboratory? (b) How far would they travel (on the average) before they decay? (c) What would your answer be to part (b) if you neglected time dilation? (d) What is the interval in spacetime between creation of a typical pion and its decay?

1.25. You have been posted to a remote region of space to monitor traffic. Near the end of a quiet shift, a spacecraft streaks past. Your laser-based measuring device reports the spacecraft's length to be 85 m. The identification transponder reports it to be the NCXXB-12, a cargo craft of proper length 100 m. In transmitting your report to headquarters, what speed should you give for this spacecraft?

1.26. The light clock in the spaceship in Figure 1-25 uses a light pulse moving up the \( y \)-axis to reflect back from a mirror as the ship moves along the \( x \)-axis. Suppose instead the light pulse moves along the \( x' \)-axis between \( x' = 0 \) and a mirror at \( x' = L \). (a) What is the time required for the pulse to make a round trip in the rest system of the spaceship? (b) What is the round-trip time in the laboratory frame? (c) Does the result in (b) agree with that expected from time dilation? Justify your answer.

1.27. Two spacecrafts pass each other traveling in opposite directions. A passenger on ship \( A \), which she knows to be 100 m long, notes that ship \( B \) is moving with a speed of 0.92c relative to \( A \) and that the length of \( B \) is 36 m. What are the lengths of the two spacecrafts measured by a passenger in \( B' \)?

1.28. A meter stick at rest in \( S' \) is tilted at an angle of 30° to the \( x' \)-axis. If \( S' \) moves at \( \beta = 0.8 \), how long is the meter stick as measured in \( S \) and what angle does it make with the \( x \)-axis? 1.29. A rectangular box at rest in \( S' \) has sides \( a' = 2 \, \text{m}, \ b' = 2 \, \text{m}, \ c' = 4 \, \text{m} \) and is oriented as shown in Figure 1-44. \( S' \) moves with \( \beta = 0.65 \) with respect to the laboratory frame \( S \). (a) Compute the volume of the box in \( S' \) and in \( S \). (b) Draw an accurate diagram of the box as seen by an observer in \( S \).

### Section 1-5 The Doppler Effect

1.30. How fast must you be moving toward a red light (\( \lambda = 650 \, \text{nm} \)) for it to appear yellow (\( \lambda = 590 \, \text{nm} \))? green (\( \lambda = 525 \, \text{nm} \))? blue (\( \lambda = 460 \, \text{nm} \))? 1.31. A distant galaxy is moving away from us at speed \( 1.85 \times 10^7 \, \text{m/s} \). Calculate the fractional red shift \( (\lambda' - \lambda) / \lambda \) of the light from this galaxy.

1.32. The light from a nearby star is observed to be shifted toward the blue by 2 percent, i.e., \( f_{\text{obs}} = 1.02 f_0 \). Is the star approaching or receding from Earth? How fast is it moving? (Assume motion is directly toward or away from Earth to avoid superluminal speeds.)

1.33. Stars typically emit the red light of atomic hydrogen with wavelength 656.3 nm (called the \( \text{H}_\alpha \) spectral line). Compute the wavelength of that light observed at Earth from stars receding directly from us with relative speed \( v = 10^{-2}c \), \( v = 10^{-3}c \), and \( v = 10^{-4}c \).

### Section 1-6 The Twin Paradox and Other Surprises

1.34. Heide boards a spaceship and travels away from Earth at a constant velocity 0.45c toward Betelgeuse (a red giant star in the constellation Orion). One year later on Earth clocks, Heide's twin, Hans, boards a second spaceship and follows her at a constant velocity of 0.95c in the same direction. (b) Which one occurs first?

1.35. You are flying in a spaceship on a straight path at 0.8c relative to Earth. At what radial distance from Earth will you be when the spaceship streak across the face of the Sun? If you were also flying at 0.9c, how far apart would you be between now and when you fly past the Sun? 1.36. A clock in the spaceship is running at a time interval of 1 s. At what time interval will a clock on Earth measure the same interval? 1.37. Einstein's theory of relativity permits the existence of a fastest object, but we are not yet capable of creating one. Why? 1.38. In the Michelson-Morley experiment a beam of light is reflected back and forth between two mirrors. Two observers have noted that the apparatus generated an interference pattern that shifted to the left once and then to the right once each time it was used, and each plane mirror was moved 90°. Einstein predicted that the apparatus would not work, but the apparatus worked in reality. 1.39. "Einstein's relativity" implies that the speed of light is constant (e.g., from one place to another).

1.40. A friend claims to have seen an alien ship zoom by at 4 \( c \cdot y \) away. How fast did he go? 1.41. A clock starts running at 0.5c after the spaceship passes by an object at rest. (b) Is it permitted? 1.42. In the Michelson-Morley experiment, (a) must a relationship be found that relates distance by \( c \), speed, and time? 1.43. A lens has focal length \( f = 0.2 \, \text{m} \) and focal length \( \beta = 0.92 \). (a) Find the mean lifetime of a particle accelerated to a speed of 0.92c. 1.44. A beam of light is reflected by a mirror and returns to the source. (b) What is the burst continuing of the beam tube? 1.45. H. A. Michelson. How did he use the interferometer? He thought that the properties of these meters were...
same direction. (a) When Hans catches up to Heide, what will be the difference in their ages? (b) Which twin will be older?

1-35. You point a laser flashlight at the Moon, producing a spot of light on the Moon’s surface. At what minimum angular speed must you sweep the laser beam in order for the light spot to streak across the Moon’s surface with speed \(v > c\)? Why can’t you transmit information between research bases on the Moon with the flying spot?

1-36. A clock is placed in a satellite that orbits Earth with a period of 108 min. (a) By what time interval will this clock differ from an identical clock on Earth after 1 year? (b) How much time will have passed on Earth when the two clocks differ by 1.0 s? (Assume special relativity applies and neglect general relativity.)

1-37. Einstein used trains for a number of relativity thought experiments since they were the fastest objects commonly recognized in those days. Let’s consider a train moving at 0.65c along a straight track at night. Its headlight produces a beam with an angular spread of 60° according to the engineer. If you are standing alongside the track (rails are 1.5 m apart), how far from you is the train when you see its approaching headlight suddenly disappear?

**Level II**

1-38. In 1971 four portable atomic clocks were flown around the world in jet aircraft, two eastbound and two westbound, to test the time dilation predictions of relativity. (a) If the westbound plane flew at an average speed of 1500 km/h relative to the surface, how long would it have to fly for the clock on board to lose 1 second relative to the reference clock on the ground at the U.S. Naval Observatory? (b) In the actual experiment the planes circumflew Earth once and the observed discrepancy of the clocks was 273 ns. What was the average speed of each plane?

1-39. “Ether drag” was among the suggestions made to explain the null result of the Michelson-Morley experiment (see More section). The phenomenon of stellar aberration refutes this proposal. Suppose Earth moves relative to the ether at velocity \(v\) and a light beam (e.g., from a star) approaches Earth at an angle \(\theta\) with respect to \(v\). (a) Show that the angle of approach in Earth’s reference frame \(\theta’\) is given by

\[
\tan \theta’ = \frac{\sin \theta}{\cos \theta + v/c}
\]

(b) \(\theta’\) is the stellar aberration angle. If \(\theta = 90^\circ\), by how much does \(\theta’\) differ from 90°?

1-40. A friend of yours who is the same age as you travels to the star Alpha Centauri, which is 4 \(c \cdot y\) away, and returns immediately. He claims that the entire trip took just 6 years. (a) How fast did he travel? (b) How old are you when he returns? (c) Draw a spacetime diagram that verifies your answers to (a) and (b).

1-41. A clock is placed in a satellite that orbits Earth with a period of 90 min. By what time interval will this clock differ from an identical clock on Earth after 1 year? (Assume that special relativity applies.)

1-42. In frame \(S\), event \(B\) occurs 2 \(\mu s\) after event \(A\) and at \(\Delta x = 1.5\) km from event \(A\). (a) How fast must an observer be moving along the +\(x\) axis so that events \(A\) and \(B\) occur simultaneously? (b) Is it possible for event \(B\) to precede event \(A\) for some observer? (c) Draw a spacetime diagram that illustrates your answers to (a) and (b). (d) Compute the spacetime interval and proper distance between the events.

1-43. A burst of \(\pi^+\) mesons travels down an evacuated beam tube at Fermilab moving at \(\beta = 0.92\) with respect to the laboratory. (a) Compute \(\gamma\) for this group of pions. (b) The proper mean lifetime of pions is \(2.6 \times 10^{-8}\) s. What mean lifetime is measured in the lab? (c) If the burst contained 50,000 pions, how many remain after the group has traveled 50 m down the beam tube? (d) What would be the answer to (c) ignoring time dilation?

1-44. H. A. Lorentz suggested 15 years before Einstein’s 1905 paper that the null effect of the Michelson-Morley experiment could be accounted for by a contraction of that arm of the interferometer lying parallel to Earth’s motion through the ether to a length \(L = L_0(1 - \beta^2/c^2)^{-1/2}\). He thought of this, incorrectly, as an actual shrinking of matter. By about how many atomic diameters would the material in the parallel arm of the interferometer have to shrink in order
to account for the absence of the expected shift of 0.4 of a fringe width? (Assume the diameter of atoms to be about $10^{-6}$ m.)
1-45. Observers in reference frame $S$ see an explosion located at $x_1 = 480$ m. A second explosion occurs 5 μs later at $x_2 = 1200$ m. In reference frame $S'$, which is moving along the $+x$ axis at speed $\nu$, the explosions occur at the same point in space. (a) Draw a spacetime diagram describing this situation. (b) Determine $\nu$ from the diagram. (c) Calibrate the $ct'$ axis and determine the separation in time in μs between the two explosions as measured in $S'$. (d) Verify your results by calculation.
1-46. Two spaceships, each 100 m long when measured at rest, travel toward each other with speeds of 0.85c relative to Earth. (a) How long is each ship as measured by someone on Earth? (b) How fast is each ship traveling as measured by an observer on the other? (c) How long is one ship when measured by an observer on the other? (d) At time $t = 0$ on Earth, the fronts of the ships are together as they just begin to pass each other. At what time on Earth are their ends together? (e) Sketch accurately scaled diagrams in the frame of one of the ships showing the passing of the other ship.
1-47. If $\nu$ is much less than $c$, the Doppler frequency shift is approximately given by $\Delta f / f_0 = \pm \beta$, both classically and relativistically. A radar transmitter-receiver bounces a signal off an aircraft and observes a fractional increase in the frequency of $\Delta f / f_0 = 8 \times 10^{-7}$. What is the speed of the aircraft? (Assume the aircraft to be moving directly toward the transmitter.)
1-48. The null result of the Michelson-Morley experiment could be explained if the speed of light depended on the motion of the source relative to the observer. Consider a binary eclipsing star system, that is, a pair of stars orbiting their common center of mass with Earth lying in the orbital plane of the system, as is very nearly the case for the binary system Algol (see More section about the Michelson-Morley experiment). Assume that the stars in the system have circular orbits with a period of 115 days and that one of the stars’ orbital speed is 32 km/s (about the same as Earth’s orbital speed around the Sun). If the suggestion above were true, astronomers would simultaneously see two images of the star in opposition, i.e., on opposite sides of its orbit. What is the minimum distance $L$ from Earth to the binary for this phenomenon to occur?
1-49. Frames $S$ and $S'$ are moving relative to each other along the $x$ and $x'$ axes. They set their clocks to $t = t' = 0$ when their origins coincide. In frame $S$, event 1 occurs at $x_1 = 1 c \cdot y$ and $t_1 = 1$ and event 2 occurs at $x_2 = 2.0 c \cdot y$ and $t_2 = 0.5 y$. These events occur simultaneously in frame $S'$. (a) Find the magnitude and direction of the velocity of $S'$ relative to $S$. (b) At what time do both of these events occur as measured in $S'$? (c) Compute the spacetime interval $\Delta s$ between the events. (d) Is the interval spacelike, timelike, or lightlike? (e) What is the proper distance $L'$ between the events?
1-50. Do Problem 1-49 parts (a) and (b) using a spacetime diagram.
1-51. An observer in frame $S$ standing at the origin observes two flashes of colored light separated spatially by $\Delta x = 2400$ m. A blue flash occurs first, followed by a red flash 5 μs later. An observer in $S'$ moving along the $x$ axis at speed $\nu$ relative to $S$ also observes the flashes 5 μs apart and with a separation of 2400 m, but the red flash is observed first. Find the magnitude and direction of $\nu$.
1-52. A cosmic-ray proton streaks through the lab with velocity 0.85c at an angle of $50^\circ$ with the $-x$ direction (in the $xy$ plane of the lab). Compute the magnitude and direction of the proton’s velocity when viewed from frame $S'$ moving with $\beta = 0.72$.

Level III

1-53. A meter stick is parallel to the $x$ axis in $S$ and is moving in the $+y$ direction at constant speed $\nu$. From the viewpoint of $S'$ show that the meter stick will appear tilted at an angle $\theta'$ with respect to the $x'$ axis of $S'$ moving in the $+x$ direction at $\beta = 0.65$. Compute the angle $\theta'$ measured in $S'$.
1-54. The equation for the spherical wave front of a light pulse that begins at the origin at time $t = 0$ is $x^2 + y^2 + z^2 - (ct)^2 = 0$. Using the Lorentz transformation, show that such a light pulse also has a spherical wave front in $S'$ by showing that $x'^2 + y'^2 + z'^2 - (ct')^2 = 0$ in $S'$.
1-55. An interesting paradox has been suggested by R. Shaw that goes like this. A very thin steel plate with a circular hole 1 m in diameter centered on the $y$ axis lies parallel to the $xz$ plane in frame $S$. A meter stick $D$ is carried from the laboratory by an observer in frame $S$. Since the meter stick is 1 m long, the observer is moving at $\beta \approx 0.7$ and the meter stick is to be contracted by Lorentz contraction to a length $D'$.<br>1-56. Two observers, A and B, are in frame $S$, the Lorenz frames are $S$ and $S'$, and $\Delta x$ is the time in frame $S$ separating the events of $x_1$ to $x_2$ in frame $S$. Suppose that the event at $x_1$ to $x_2$ in frame $S$ is 10 μs.
1-57. Two observers in frame $S$, the time in frame $S'$ of the events of $x_1$ to $x_2$ is 10 μs. Let $\alpha$ be the angle of $S'$ relative to $S$. Find $\alpha$.
1-58. The center of the Earth is 6371 km from the center of the Sun. If the Earth were at rest, how long would it take for a clock to drift 90° relative to the Sun, assuming that the Sun drifts relative to the Earth? (Include relativistic factors.)
1-59. Two observers A and B are in the laboratory frames $S$ and $S'$, respectively. (a) Describe the opposite directions of rotation of $S'$ and $S$. (b) Determine the vector between the centers of $S'$ and $S$ in $S$. (c) Using the Lorentz transformation, determine the interval $\Delta s$ in $S'$ between two events $x_1$ and $x_2$ in $S$. Compare this result with the interval $\Delta s$ in $S$ and find the factor between $\Delta s$ and $\Delta s'$. (d) Using the Lorentz transformation, determine the intermediate points $x_1'$ and $x_2'$ of an event $x$ in $S$. Compare this result with the intermediate points $x_1''$ and $x_2''$ of an event $x''$ in $S$ and find the factor between $x_1''$ and $x_2''$.
1-60. Suppose a spaceship is traveling at near light speed, e.g., the Earth at a position in the solar system at high speed.
in frame $S$ and moves in the $+y$ direction at constant speed $v$, as illustrated in Figure 1-45. A meter stick lying on the $x$ axis moves in the $+x$ direction with $\beta = v/c$. The steel plate arrives at the $y = 0$ plane at the same instant that the center of the meter stick reaches the origin of $S$. Since the meter stick is observed in $S$ to be contracted, it passes through the $1\text{-m}$ hole in the plate with no problem. A paradox appears to arise when one considers that an observer in $S'$, the rest system of the meter stick, measures the diameter of the hole in the plate to be contracted in the $x$ dimension and, hence, becomes too small to pass the meter stick, resulting in a collision. Resolve the paradox. Will there be a collision?

1-56. Two events in $S$ are separated by a distance $D = x_2 - x_1$ and a time $T = t_2 - t_1$. (a) Use the Lorentz transformation to show that in frame $S'$, which is moving with speed $v$ relative to $S$, the time separation is $t_2' - t_1' = \gamma(T - vt/c^2)$. (b) Show that the events can be simultaneous in frame $S'$ only if $D$ is greater than $cT$. (c) If one of the events is the cause of the other, the separation $D$ must be less than $cT$ since $D/c$ is the smallest time that a signal can take to travel from $x_1$ to $x_2$ in frame $S$. Show that if $D$ is less than $cT$, $t_2'$ is greater than $t_1'$ in all reference frames. (d) Suppose that a signal could be sent with speed $c^2 > c$ so that in frame $S$ the cause precedes the effect by the time $T = D/c'$. Show that there is then a reference frame moving with speed $v$ less than $c$ in which the effect precedes the cause.

1-57. Two observers agree to test time dilation. They use identical clocks and one observer in frame $S'$ moves with speed $v = 0.6c$ relative to the other observer in frame $S$. When their origins coincide, they start their clocks. They agree to send a signal when their clocks read 60 min and to send a confirmation signal when each receives the other’s signal. (a) When does the observer in $S$ receive the first signal from the observer in $S'$? (b) When does he receive the confirmation signal? (c) Make a table showing the times in $S$ when the observer sent the first signal, received the first signal, and received the confirmation signal. How does this table compare with one constructed by the observer in $S'$?

1-58. The compact disk in a CD-ROM drive rotates with angular speed $\omega$. There is a clock at the center of the disk and one at a distance $r$ from the center. In an inertial reference frame, the clock at distance $r$ is moving with speed $u = r\omega$. Show that from time dilation in special relativity, time intervals $\Delta t_o$ for the clock at rest and $\Delta t_r$ for the moving clock are related by

$$\frac{\Delta t_r - \Delta t_o}{\Delta t_o} = \frac{r^2\omega^2}{2c^2}, \quad \text{if} \quad r\omega \ll c$$

1-59. Two rockets, $A$ and $B$, leave a space station with velocity vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ relative to each other. (a) Determine the velocity of $A$ relative to $B$, $\mathbf{v}_{AB}$. (b) Determine the velocity of $B$ relative to $A$, $\mathbf{v}_{BA}$. (c) Explain why $\mathbf{v}_{AB}$ and $\mathbf{v}_{BA}$ do not point in opposite directions.

1-60. Suppose a system $S$ consisting of a cubic lattice of meter sticks and synchronized clocks, e.g., the eight clocks closest to you in Figure 1-13, moves from left to right (the $+x$ direction) at high speed. The meter sticks parallel to the $x$ direction are, of course, contracted and the cube
would be measured by an observer in a system $S'$ to be foreshortened in that direction. However, recalling that your eye constructs images from light waves that reach it simultaneously, not those leaving the source simultaneously, sketch what your eye would see in this case. Scale contractions and show any angles accurately. (Assume the moving cube to be farther than 10 m from your eye.)

1-61. Figure 1-11b (in the More section about the Michelson-Morley experiment) shows an eclipsing binary. Suppose the period of the motion is $T$ and the binary is a distance $L$ from Earth, where $L$ is sufficiently large so that points $A$ and $B$ in Figure 1-11b are a half orbit apart. Consider the motion of one of the stars and $(a)$ show that the star would appear to move from $A$ to $B$ in time $T/2 + 2L/v(\gamma^2 \cos^2 \theta - v^2)$ and from $B$ to $A$ in time $T/2 - 2L/v(\gamma^2 - v^2)$, assuming classical velocity addition applies to light, i.e., that emission theories of light were correct. $(b)$ What rotational period would cause the star to appear to be at both $A$ and $B$ simultaneously?$(c)$ Show that if a particle moves at an angle $\theta$ with respect to the $x$ axis with speed $u$ in system $S$, it moves at an angle $\theta'$ with the $x'$ axis in $S'$ given by

$$\tan \theta' = \frac{\sin \theta}{\gamma(\cos \theta - v/u)}$$

1-62. Like jets emitted from some galaxies (see Figure 1-41), some distant astronomical objects can appear to travel at speeds greater than $c$ across our line of sight. Suppose distant galaxy AB15 moving with velocity $v$ at an angle $\theta$ with respect to the direction toward Earth emits two bright flashes of light separated by time $\Delta t$ on the galaxy AB15 local clock. Show that $(a)$ the time interval $\Delta t_{\text{Earth}} = \Delta t(1 - \beta \cos \theta)$ and $(b)$ the apparent speed of AB15 measured by observers on Earth is $v_{\text{app}} = \frac{\Delta x_{\text{Earth}}}{\Delta t_{\text{Earth}}} = \frac{\beta \sin \theta}{1 - \beta \cos \theta}$. $(c)$ For $\beta = 0.75$, compute the value of $\theta$ for which $v_{\text{app}} = c$.

In the opening example, Newton’s laws specify that $F/m$ is the only force acting on a body in an inertial frame. It is this observation that leads to the Lorentz transformation between inertial frames. If a particle moves with speed $v$ in one inertial frame, the Lorentz transformation predicts it moves with speed $\beta v$ in another inertial frame.

Thus, $F/m$ does not hold in a noninertial frame.

It is reassuring to see that Newton’s laws imply $F/m$ acts for a local body in every inertial frame. In noninertial frames, $\gamma$ becomes greater than 1, so that $\beta$ becomes greater than 1. If a particle moves faster than the speed of light in a noninertial frame, it violates the second postulate of special relativity.

In this chapter, we will discuss how special relativity replaces classical mechanics.