Lecture 18

Agenda:
1. Review for exam
2. Assignment: For Monday, Read chapter 14

Newton’s Laws

Three blocks are connected on the table as shown. The table has a coefficient of kinetic friction of 0.350, the masses are \( m_1 = 4.00 \text{ kg} \), \( m_2 = 1.00 \text{kg} \) and \( m_3 = 2.00 \text{kg} \). Now the pulleys are uniform disks of 4.00 kg mass and 0.50 m radius. The rope does not slip.

![Diagram of three blocks connected on a table with a coefficient of kinetic friction of 0.350, masses \( m_1 = 4.00 \text{ kg} \), \( m_2 = 1.00 \text{kg} \), and \( m_3 = 2.00 \text{kg} \), and pulleys with a mass of 4.00 kg and a radius of 0.50 m.]

a) If \( m_3 \) starts from rest how fast is it going after it goes up 2.0 m
Work, Energy & Circular Motion

A mass, 11 kg, slides down of a frictionless circular path of radius, 5.0 m, as shown in the figure. Initially it moves only vertically and, at the end, only horizontally (1/4 of a circle all told). Gravity, 10 m/s$^2$, acts along the vertical.

If the initial velocity is 2 m/s downward then

(a) What is the work done by gravity on the mass?

$$W = mgR = 11 \times 10 \times 5 = 550 \text{ J}$$

(b) What is the final speed of the mass when it reaches the bottom?

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgR = 22 \text{ J} + 550 \text{ J} = 572 \text{ J}$$

$$v_f = \left( \frac{1144}{11} \right)^{\frac{1}{2}} \text{ m/s}$$
Work, Energy & Circular Motion

A mass, 11 kg, slides down of a frictionless circular path of radius, \(5.0 \text{ m}\), as shown in the figure. Initially it moves only vertically and, at the end, only horizontally (1/4 of a circle all told). Gravity, \(10 \text{ m/s}^2\), acts along the vertical.

If the initial velocity is 2 m/s downward then

(c) What is the normal force on the mass when it reaches the bottom

\[
\Sigma F_y = m a_c = N - mg = m \frac{v^2}{R}
\]

\[
N = mg + m \frac{v^2}{R} = (110 + 11 \times \frac{1144}{11} / 5) \text{ N}
\]

\[
= 340 \text{ N}
\]

Momentum and collisions

Remember vector components

A 5 kg cart rolling without friction to the right at 10 m/s collides and sticks to a 5 kg motionless block on a 30° frictionless incline.

How far along the incline do the joined blocks slide?
Momentum and collisions

1. Remember vector components

\[ m v_i \cos \theta, \quad m v_f \]

A 5 kg cart rolling without friction to the right at 10 m/s collides and sticks to a 5 kg motionless block on a \( \theta = 30^\circ \) frictionless incline.

1. Momentum parallel to incline is conserved

1. Normal force (by ground on cart) is \( \perp \) to the incline

\[ m v_i \cos \theta + m 0 = 2m v_f \]

\[ v_f = \frac{v_i \cos \theta}{2} = 4.4 \text{ m/s} \]

Now use work-energy

\[ 2mgh + 0 = \frac{1}{2} 2mv^2_f \quad d = h \frac{1}{2} v^2_f / (g \sin \theta) \]

Springs

1. A Hooke’s Law spring with a spring constant of 200 N/m is first stretched 3.0 m past its equilibrium distance and then is stretched 9.0 m.

1. How much work must be done to go from 3.0 m to 9.0 m?

\[ W = U_{\text{final}} - U_{\text{initial}} = \frac{1}{2} k (x-x_{eq})_{\text{final}}^2 - \frac{1}{2} k (x-x_{eq})_{\text{init}}^2 \]

\[ = 100 \ [(9)^2 -(3)^2] \ J = 100(72) \ J = 7200 \ J \]
Chapter 7 (Newton’s 3rd Law) & Chapter 8

Newton’s Second Law
Expressed in x- and y-component form:

\[(F_{\text{net}})_x = \sum F_x = ma_x\]
\[(F_{\text{net}})_y = \sum F_y = ma_y\]

Expressed in rotational form:

\[(F_{\text{net}})_{\theta} = \sum F_{\theta} = \frac{m\omega^2}{r}\]
\[(F_{\text{net}})_{r} = \sum F_{r} = \begin{cases} 0 & \text{uniform circular motion} \\ ma_r & \text{nonuniform circular motion} \end{cases}\]

Angular velocity

\[\omega = \frac{d\theta}{dt}\]
\[\nu_{\theta} = \omega r\]

Angular acceleration

\[\alpha = \frac{d\omega}{dt}\]
\[a_r = \omega \nu_{\theta}\]

Orbits

A circular orbit has radius \(r\) if

\[v = \sqrt{rg}\]

Chapter 8

Uniform Circular Motion

- \(v\) is constant.
- \(\vec{F}_{\text{net}}\) points toward the center of the circle.
- The centripetal acceleration \(\vec{a}\) points toward the center of the circle. It changes the particle’s direction but not its speed.

Nonuniform Circular Motion

- \(v\) changes.
- \(\vec{a}\) is parallel to \(\vec{F}_{\text{net}}\).
- The radial component \(a_r\) changes the particle’s direction.
- The tangential component \(a_t\) changes the particle’s speed.
Chapter 9

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an isolated system is a constant. Thus

$\vec{p}_i = \vec{p}_f$

Newton’s Second Law

In terms of momentum, Newton’s second law is

$\vec{F} = \frac{d\vec{P}}{dt}$

System

A group of interacting particles.

Isolated system

A system on which there are no external forces or the net external force is zero.

Before-and-after pictorial representation

- Define the system.
- Use two drawings to show the system before and after the interaction.
- List known information and identify what you are trying to find.

Law of conservation of momentum

The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system’s total momentum.
Chapter 10

Law of Conservation of Mechanical Energy

If there are no friction or other energy-loss processes (to be explored more thoroughly in Chapter 11), then the mechanical energy $E_{\text{mech}} = K + U$ of a system is conserved. Thus

$$K_i + U_i = K_f + U_f$$

- $K$ is the sum of the kinetic energies of all particles.
- $U$ is the sum of all potential energies.

Kinetic energy is an energy of motion:

$$K = \frac{1}{2}mv^2$$

Potential energy is an energy of position

- Gravitational: $U_g = mgy$
- Elastic: $U_s = \frac{1}{2}k(\Delta s)^2$

Basic Energy Model

Energy into system

Energy can be transformed with the system without loss.

Mechanical energy $E_{\text{mech}} = K + U$

Energy out of system

Chapter 10

Energy diagrams

These diagrams show the potential-energy curve PE and the total mechanical energy line TE.

- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a turning point.
- Minima in the PE curve are points of stable equilibrium.
- Maxima are points of unstable equilibrium.

Hooke’s law

The restoring force of an ideal spring is

$$F_R = -k\Delta s$$

where $k$ is the spring constant and $\Delta s = s - s_e$ is the displacement from equilibrium.

Basic Energy Model

- Energy is transferred to or from the system by work.
- Energy is transformed within the system.

Two versions of the energy equation are

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}}$$

$$K_i + U_i + \Delta E_{\text{int}} = K_f + U_f + W_{\text{ext}}$$
Chapter 10

Law of Conservation of Energy

- Isolated system: $W_{\text{ext}} = 0$. The total energy $E_{\text{sys}} = E_{\text{mech}} + E_{\text{th}}$ is conserved. $\Delta E_{\text{sys}} = 0$.
- Isolated, nondissipative system: $W_{\text{ext}} = 0$ and $W_{\text{diss}} = 0$. The mechanical energy $E_{\text{mech}}$ is conserved. $\Delta E_{\text{mech}} = 0$ or $K_i + U_i = K_f + U_f$

The work-kinetic energy theorem is

$$\Delta K = W_{\text{net}} = W_c + W_{\text{diss}} + W_{\text{ext}}$$

With $W_c = -\Delta U$ for conservative forces and $W_{\text{diss}} = -\Delta E_{\text{th}}$ for dissipative forces, this becomes the energy equation.

Chapter 11

The work done by a force on a particle as it moves from $s_i$ to $s_f$ is

$$W = \int_{s_i}^{s_f} F_i \, ds = \text{area under the force curve}$$

$$= \vec{F} \cdot \Delta \vec{s} \text{ if } \vec{F} \text{ is a constant force}$$

Conservative forces are forces for which the work is independent of the path followed. The work done by a conservative force can be represented as a potential energy:

$$\Delta U = U_f - U_i = -W_{\text{cons}}(i \rightarrow f)$$

A conservative force is found from the potential energy by

$$F_i = -\frac{dU}{ds} = \text{negative of the slope of the PE curve}$$

Dissipative forces transform macroscopic energy into thermal energy, which is the microscopic energy of the atoms and molecules. For friction:

$$\Delta E_{\text{th}} = f_s \Delta s$$
Chapter 11

Power is the rate at which energy is transferred or transformed:

\[ P = \frac{dE_{\text{sys}}}{dt} \]

For a particle moving with velocity \( \vec{v} \), the power delivered to the particle by force \( \vec{F} \) is \( P = \vec{F} \cdot \vec{v} = F_v \cos \theta \).

Dot product

\[ \vec{A} \cdot \vec{B} = AB \cos \alpha = A_x B_x + A_y B_y \]

Chapter 12

The moment of inertia and Center of Mass

The moment of inertia is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If \( I_{\text{cm}} \) is known, the \( I \) about a parallel axis distance \( d \) away is given by the parallel-axis theorem: \( I = I_{\text{cm}} + Md^2 \).

Rotational Dynamics

Every point on a rigid body rotating about a fixed axis has the same angular velocity \( \omega \) and angular acceleration \( \alpha \).

Newton’s second law for rotational motion is

\[ \alpha = \frac{\tau_{\text{net}}}{I} \]

Use rotational kinematics to find angles and angular velocities.
Example problem: Going in circles

1. A 2.0 kg disk tied to a 0.50 m string undergoes circular motion on a rough but horizontal table top. The kinetic coefficient of friction is 0.25. If the disk starts out at 5.0 rev/sec how many revolutions does it make before it comes to rest?

   - Work-energy theorem
   - \( W = F \cdot d = 0 - \frac{1}{2} m v^2 \)
   - \( F = -\mu mg \cdot d = - \frac{1}{2} m v^2 \)
   - \( d = \frac{v^2}{(2\mu g)} = \frac{(5.0 \times 2\pi \times 0.50)^2}{(0.50 \times 10)} \) m = \( 5 \pi^2 \) m
   - \( \text{Rev} = \frac{d}{2\pi r} = 16 \) revolutions

   - What if the disk were tilted by 60°?
Work and Energy

A block of mass \( m \) is connected by a spring to the ceiling. The block is held at a position where the spring is unstretched and then released. When released, the block

(a) remains at rest.

(b) oscillates about the unstretched position

(c) oscillates about a position that is lower than the unstretched position

(d) oscillates about a position that is higher than the unstretched position
Momentum & Impulse

1. A rubber ball collides head on (i.e., velocities are opposite) with a clay ball of the same mass. The balls have the same speed, \( v \), before the collision, and stick together after the collision. What is their speed immediately after the collision?

   A. 0
   B. \( \frac{1}{2} v \)
   C. 2 \( v \)
   D. 4 \( v \)
Momentum, Work and Energy

1. A 0.40 kg block is pushed up against a spring (with spring constant 270 N/m) on a frictionless surface so that the spring is compressed 0.20 m. When the block is released, it slides across the surface and collides with the 0.60 kg bob of a pendulum. The bob is made of clay and the block sticks to it. The length of the pendulum is 0.80 m. (See the diagram.)

To what maximum height above the surface will the ball/block assembly rise after the collision? (g=9.8 m/s²)

A. 2.2 cm
B. 4.4 cm
C. 11 cm
D. 22 cm
E. 44 cm
F. 55 cm
Work and Energy

A mass is attached to a Hooke’s law spring on a horizontal surface as shown in the diagram below. When the spring is at its natural length, the block is at position Y.

When released from position X, how will the spring potential energy vary as the block moves from X to Y to Z?

(a) It will steadily increase from X to Z.
(b) It will steadily decrease from X to Z.
(c) It will increase from X to Y and decrease from Y to Z.
(d) It will decrease from X to Y and increase from Y to Z.
Work and Energy

An object moves along a line under the influence of a single force. The force vector is parallel to the displacement. The area under the force vs. position graph represents

(a) the impulse delivered to the object

(b) the work done on the object.

(c) the change in the velocity of the object.

(d) the momentum of the object.
Momentum and Impulse

Henri Lamothe holds the world record for the highest shallow dive. He belly-flopped from a platform 12.0 m high into a tank of water just 30.0 cm deep! Assuming that he had a mass of 50.0 kg and that he stopped just as he reached the bottom of the tank, what is the magnitude of the impulse imparted to him while in the tank of water (in units of kg m/s)?

(a) 121  
(b) 286  
(c) 490  
(d) 623  
(e) 767

\[ \Delta p = \sqrt{2 \times 9.8 \times 12.3} \times 50 \]
Two particles, one positively charged and one negatively charged, are held apart. Since oppositely charged objects attract one another, the particles will accelerate towards each other when released. Let \( W^+ \) be the work done on the positive charge by the negative charge. Let \( W^- \) be the work done on the negative charge by the positive charge. While the charges are moving towards each other, which of the following statements is correct?

(a) \( W^+ \) is positive and \( W^- \) is negative.
(b) \( W^+ \) is negative and \( W^- \) is positive.
(c) Both \( W^+ \) and \( W^- \) are positive.
(d) Both \( W^+ \) and \( W^- \) are negative.
(e) Without knowing the coordinate system, the sign of the work can not be determined.
Momentum & Impulse

Suppose that in the previous problem, the positively charged particle is a proton and the negatively charged particle, an electron. (The mass of a proton is approximately 1,840 times the mass of an electron.) Suppose that they are released from rest simultaneously. If, after a certain time, the change in momentum of the proton is $\Delta p$, what is the magnitude of the change in momentum of the electron?

(a) $\Delta p / 1840$
(b) $\Delta p$
(c) $1840 \Delta p$
Work and Energy

A block slides along a frictionless surface before colliding with a spring. The block is brought momentarily to rest by the spring after traveling some distance. The four scenarios shown in the diagrams below are labeled with the mass of the block, the initial speed of the block, and the spring constant.

Rank the scenarios in order of the distance the block travels, listing the largest distance first.

(a) B, A, C = D
(b) B, C, A, D
(c) B, C = D, A
(d) C = B, A, D
(e) C = B = D, A
Newton’s Laws

1. Two boxes are connected to each other as shown. The system is released from rest and the 1.00 kg box falls through a distance of 1.00 m. The surface of the table is frictionless. What is the kinetic energy of box B just before it reaches the floor? \( g = 9.81 \text{ m/s}^2 \)

(a) 2.45 J
(b) 4.90 J
(c) 9.80 J
(d) 9.24 J
(e) 9.32 J

Work and Energy

1. If it takes 5.35 J of work to stretch a Hooke’s law spring 12.2 cm from its un-stretched length, how much work is required to stretch an identical spring by 17.2 cm from its un-stretched length?

(a) 0.90 J
(b) 5.3 J
(c) 7.2 J
(d) 10.6 J
(e) 11.0 J
Work and Energy

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(b) 5.3 J
(c) 7.2 J
(d) 10.6 J
(e) 11.0 J

Work and Forces

1. A 25.0 kg chair is pushed 2.00 m at constant speed along a horizontal surface with a constant force acting at 30.0 degrees below the horizontal. If the friction force between the chair and the surface is 55.4 N, what is the work done by the pushing force?

(a) 85 J
(b) 98 J
(c) 111 J
(d) 113 J
(e) 128 J
**Work and Forces**

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**Work and Power**

A 100 kg elevator is carrying 6 people, each weighing 70 kg. They all want to travel to the top floor, 75 m from the floor they entered at. How much power will the elevator motor supply to lift this in 45 seconds at constant speed?

(a) $1.2 \cdot 10^2$ W
(b) $7.0 \cdot 10^2$ W
(c) $8.7 \cdot 10^2$ W
(d) $6.9 \cdot 10^3$ W
(e) $8.5 \cdot 10^3$ W
Work and Power

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(e) $8.5 \times 10^3$ W

Conservation of Momentum

A woman is skating to the right with a speed of 12.0 m/s when she is hit in the stomach by a giant snowball moving to the left. The mass of the snowball is 2.00 kg, its speed is 25.0 m/s and it sticks to the woman's stomach. If the mass of the woman is 60.0 kg, what is her speed after the collision?

(a) 10.8 m/s

(b) 11.2 m/s

(c) 12.4 m/s

(d) 12.8 m/s
Conservation of Momentum

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(a) 10.8 m/s  
(b) 11.2 m/s  
(c) 12.4 m/s  
(d) 12.8 m/s

Momentum and Impulse

A stunt man jumps from the roof of a tall building, but no injury occurs because the person lands on a large, air-filled bag. Which one of the following statements best describes why no injury occurs?

(a) The bag provides the necessary force to stop the person.  
(b) The bag reduces the impulse to the person.  
(c) The bag reduces the change in momentum.  
(d) The bag decreases the amount of time during which the momentum is changing and reduces the average force on the person.  
(e) The bag increases the amount of time during which the momentum is changing and reduces the average force on the person.
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(e) The bag increases the amount of time during which the momentum is changing and reduces the average force on the person.

Newton’s Laws

Two sleds are hooked together in tandem. The front sled is twice as massive as the rear sled. The sleds are pulled along a frictionless surface by a force $F$, applied to the more massive sled. The tension in the rope between the sleds is $T$. Determine the ratio of the magnitudes of the two forces, $T/F$.

(a) 0.33
(b) 0.50
(c) 0.67
(d) 1.5
(e) 2.0
(f) 3.0
Momentum and Impulse

Two blocks of mass $m_1 = M$ and $m_2 = 2M$ are both sliding towards you on a frictionless surface. The linear momentum of block 1 is half the linear momentum of block 2. You apply the same constant force to both objects in order to bring them to rest. What is the ratio of the two stopping distances $d_2/d_1$?

(a) $1/2$
(b) $1/2^{1/2}$
(c) 1
(d) $2^{1/2}$
(e) 2
(f) Cannot be determined without knowing the masses of the objects and their velocities.
Work and Energy

An object is acted upon by only two forces, one conservative and one nonconservative, as it moves from point A to point B. The kinetic energy of the object at points A and B are equal if

A. the sum of the two forces' work is zero
B. the work of the nonconservative force is zero
C. the work of the conservative force is zero
D. the work of the conservative force is equal to the work of the nonconservative force
E. None of the above will make them equal
Work and Energy

A 6.0 kg block is pushed up against an ideal Hooke’s law spring (of spring constant 3750 N/m) until the spring is compressed a distance x. When it is released, the block travels along a track from one level to a higher level, by moving through an intermediate valley (as shown in the diagram). The track is frictionless until the block reaches the higher level. There is a frictional force stops the block in a distance of 1.2 m. If the coefficient of friction between the block and the surface is 0.60, what is x? (Let g = 9.81 m/s²)

(a) 0.11 m  
(b) 0.24 m  
(c) 0.39 m  
(d) 0.48 m  
(e) 0.56 m
Momentum and Impulse

In a table-top shuffleboard game, a heavy moving puck collides with a lighter stationary puck as shown. The incident puck is deflected through an angle of 20° and both pucks are eventually brought to rest by friction with the table. The impulse approximation is valid (i.e., the time of the collision is short relative to the time of motion so that momentum is conserved).

Which of the following statements is correct?
A. The collision must be inelastic because the pucks have different masses.
B. The collision must be inelastic because there is friction between the pucks and the surface.
C. The collision must be elastic because the pucks bounce off each other.
D. The collision must be elastic because, in the impulse approximation, momentum is conserved.
E. There is not enough information given to decide whether the collision is elastic or inelastic.