Lecture 14

Goals:
• Chapter 10
  v Understand the relationship between motion and energy
  v Define Kinetic Energy
  v Define Potential Energy
  v Define Mechanical Energy
  v Exploit Conservation of energy principle in problem solving
  v Understand Hooke’s Law spring potential energies
  v Use energy diagrams

Assignment:
1  HW6 due Tuesday Oct. 25th
1  For Monday: Read Ch. 11

Kinetic & Potential energies

1 Kinetic energy, $K = \frac{1}{2} mv^2$, is defined to be the large scale collective motion of one or a set of masses

1 Potential energy, $U$, is defined to be the “hidden” energy in an object which, in principle, can be converted back to kinetic energy

1 Mechanical energy, $E_{Mech}$, is defined to be the sum of $U$ and $K$

1 Others forms of energy can be constructed
Recall if a constant force over time then

\[ y(t) = y_i + v_{yi} t + \frac{1}{2} a_y t^2 \]

\[ v(t) = v_{yi} + a_y t \]

Eliminating \( t \) gives

\[ 2 a_y (y - y_i) = v_x^2 - v_{yi}^2 \]

\[ m a_y (y - y_i) = \frac{1}{2} m (v_x^2 - v_{yi}^2) \]

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**Energy (dropping a ball)**

\[-mg (y_{final} - y_{init}) = \frac{1}{2} m (v_{y_{final}}^2 - v_{y_{init}}^2)\]

A relationship between \( y - \) displacement and change in the \( y \)-speed squared

Rearranging to give initial on the left and final on the right

\[ \frac{1}{2} m v_{yi}^2 + mg y_i = \frac{1}{2} m v_{yf}^2 + mg y_f \]

We now define \( mgy = U \) as the “gravitational potential energy”
Energy (throwing a ball)

1. Notice that if we only consider gravity as the external force then the x and z velocities remain constant.
2. To \[ \frac{1}{2} m v_{yi}^2 + mgy_i = \frac{1}{2} m v_{yi}^2 + mgy_f \]
3. Add \[ \frac{1}{2} m v_{xi}^2 + \frac{1}{2} m v_{zi}^2 \text{ and } \frac{1}{2} m v_{xf}^2 + \frac{1}{2} m v_{zf}^2 \]
4. \[ \frac{1}{2} m v_i^2 + mgy_i = \frac{1}{2} m v_f^2 + mgy_f \]
5. where \[ v_i^2 = v_{xi}^2 + v_{yi}^2 + v_{zi}^2 \]

\[ \frac{1}{2} m v^2 = K \] terms are defined to be kinetic energies
(A scalar quantity of motion)

When is mechanical energy not conserved

1. Mechanical energy is **not** conserved when there is a process which can be shown to transfer energy out of a system and that energy cannot be transferred back.
Inelastic collision in 1-D: Example 1

A block of mass $M$ is initially at rest on a frictionless horizontal surface. A bullet of mass $m$ is fired at the block with a muzzle velocity (speed) $v$. The bullet lodges in the block, and the block ends up with a speed $V$.

What is the initial energy of the system?
What is the final energy of the system?
Is energy conserved?

\[
\begin{align*}
\text{before} & \quad \rightarrow \quad \text{after} \\
\end{align*}
\]

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Inelastic collision in 1-D: Example 1

What is the momentum of the bullet with speed $v$? $m\vec{v}$

What is the initial energy of the system? $\frac{1}{2}mv^2$

What is the final energy of the system? $\frac{1}{2}(m+M)V^2$

Is momentum conserved (yes)?
Is energy conserved? Examine $E_{\text{before}} - E_{\text{after}}$

\[
\begin{align*}
\frac{1}{2}mv^2 - \frac{1}{2}(m+M)V^2 & = \frac{1}{2}mv^2 - \frac{1}{2}(mv)\frac{m}{m+M}V = \frac{1}{2}mv^2 \left(1 - \frac{m}{m+M}\right)\\
\end{align*}
\]

\[
\begin{align*}
\text{before} & \quad \rightarrow \quad \text{after} \\
\end{align*}
\]

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Elastic vs. Inelastic Collisions

1. A collision is said to be *inelastic* when “mechanical” energy (\( K + U \)) is not conserved before and after the collision.
2. How, if no net Force then momentum will be conserved.
   \[
   K_{\text{before}} + U \neq K_{\text{after}} + U
   \]
3. E.g. car crashes on ice: Collisions where objects stick together

1. A collision is said to be *perfectly elastic* when both energy & momentum are conserved before and after the collision.
   \[
   K_{\text{before}} + U = K_{\text{after}} + U
   \]
2. Carts colliding with a perfect spring, billiard balls, etc.

Energy

1. If only “conservative” forces are present, then the mechanical energy of a system is conserved

For an object acted on by gravity

\[
\frac{1}{2} m v_i^2 + mgy_i = \frac{1}{2} m v_f^2 + mgy_f
\]

\[
E_{\text{mech}} = K + U = \text{constant}
\]

\( E_{\text{mech}} \) is called “mechanical energy”

\( K \) and \( U \) may change, \( K + U \) remains a fixed value.
Example of a conservative system: The simple pendulum.

Suppose we release a mass \( m \) from rest a distance \( h_1 \) above its lowest possible point.

\( \checkmark \) What is the maximum speed of the mass and where does this happen?

\( \checkmark \) To what height \( h_2 \) does it rise on the other side?

Example: The simple pendulum.

\( \checkmark \) What is the maximum speed of the mass and where does this happen?

\[ E = K + U = \text{constant} \] and so \( K \) is maximum when \( U \) is a minimum.
Example: The simple pendulum.

What is the maximum speed of the mass and where does this happen?

\[ E = K + U = \text{constant and so } K \text{ is maximum when } U \text{ is a minimum} \]

\[ E = mgh_1 \text{ at top} \]

\[ E = mgh_1 = \frac{1}{2} mv^2 \text{ at bottom of the swing} \]

To what height \( h_2 \) does it rise on the other side?

\[ E = K + U = \text{constant and so when } U \text{ is maximum again (when } K = 0) \text{ it will be at its highest point.} \]

\[ E = mgh_1 = mgh_2 \text{ or } h_1 = h_2 \]
Potential Energy, Energy Transfer and Path

1. A ball of mass m, initially at rest, is released and follows three different paths. All surfaces are frictionless.
   1. The ball is dropped
   2. The ball slides down a straight incline
   3. The ball slides down a curved incline

After traveling a vertical distance $h$, how do the three speeds compare?

(A) $1 > 2 > 3$     (B) $3 > 2 > 1$    (C) $3 = 2 = 1$  (D) Can’t tell

Example
The Loop-the-Loop … again

1. To complete the loop the loop, how high do we have to let the release the car?
2. Condition for completing the loop the loop: Circular motion at the top of the loop ($a_c = v^2 / R$)
3. Exploit the fact that $E = U + K = constant$ ! (frictionless)

Recall that “g” is the source of the centripetal acceleration and N just goes to zero is the limiting case.
Also recall the minimum speed at the top is $v = \sqrt{gR}$
Example
The Loop-the-Loop ... again

1. Use $E = K + U = \text{constant}$
2. $mg h + 0 = mg 2R + \frac{1}{2} m v^2$
   $mg h = mg 2R + \frac{1}{2} mgR = \frac{5}{2} mgR$

\[ h = \frac{5}{2} R \]

Variable force devices: Hooke’s Law Springs

1. Springs are everywhere,

   \[ F_s = - k \Delta s \]

   $\Delta s$ is the amount the spring is stretched or compressed from its resting position.
**Exercise Hooke’s Law**

What is the spring constant “k”?

(A) 50 N/m  (B) 100 N/m  (C) 400 N/m  (D) 500 N/m

**F vs. Δx relation for a foot arch:**

![Diagram of F vs. Δx relation for a foot arch]
**Force vs. Energy for a Hooke’s Law spring**

1. \( F = -k(x - x_{\text{equilibrium}}) \)
2. \( F = ma = m \frac{dv}{dt} \)
   - \( = m \left( \frac{dv}{dx} \frac{dx}{dt} \right) \)
   - \( = m \frac{dv}{dx} v \)
   - \( = mv \frac{dv}{dx} \)
3. So \( -k(x - x_{\text{equilibrium}}) \frac{dx}{dt} = mv \frac{d^2v}{dx^2} \)
4. Let \( u = x - x_{\text{eq.}} \) & \( du = dx \)

\[
-\frac{1}{2} ku^2 \bigg|_{u_i}^{u_f} = \frac{1}{2} mv_i^2 \bigg|_{v_f}^{v_i} \\
-\frac{1}{2} ku_i^2 + \frac{1}{2} ku_f^2 = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2
\]

\[
\frac{1}{2} ku_i^2 + \frac{1}{2} mv_i^2 = \frac{1}{2} ku_f^2 + \frac{1}{2} mv_f^2
\]

**Energy for a Hooke’s Law spring**

\[
\frac{1}{2} ku_i^2 + \frac{1}{2} mv_i^2 = \frac{1}{2} ku_f^2 + \frac{1}{2} mv_f^2
\]

1. Associate \( \frac{1}{2} ku^2 \) with the “potential energy” of the spring

\[
U_{si} + K_i = U_{sf} + K_f
\]

1. Ideal Hooke’s Law springs are conservative so the mechanical energy is constant
Energy diagrams

In general:

\[ u = x - x_{eq} \]

Spring/Mass system

Ball falling

Equilibrium

Example

Spring: \( F_x = 0 \Rightarrow \frac{dU}{dx} = 0 \) for \( x = x_{eq} \)

The spring is in equilibrium position

In general: \( \frac{dU}{dx} = 0 \) for ANY function establishes equilibrium

stable equilibrium

unstable equilibrium
Comment on Energy Conservation

1. We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
   - Mechanical energy is lost:
     - Heat (friction)
     - Bending of metal and deformation

1. Kinetic energy is not conserved by these non-conservative forces occurring during the collision!

1. Momentum along a specific direction is conserved when there are no external forces acting in this direction.
   - In general, easier to satisfy conservation of momentum than energy conservation.

Comment on Energy Conservation

1. We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
   - Mechanical energy is lost:
     - Heat (friction)
     - Deformation (bending of metal)

1. Mechanical energy is not conserved when non-conservative forces are present!

1. Momentum along a specific direction is conserved when there are no external forces acting in this direction.
   - Conservation of momentum is a more general result than mechanical energy conservation.