Lecture 10

Today:

- Review session

Assignment: For Monday, Read through Chapter 8
There will be a reading quiz posted at Mastering Physics.

Exam Thursday, Oct. 6th from 7:15-8:45 PM Chapters 1-6, 7
One 8½ X 11 hand written note sheet and a calculator (for trig.)

Textbook Chapters

1. Chapter 1  Concept of Motion
2. Chapter 2  1D Kinematics
3. Chapter 3  Vector and Coordinate Systems
4. Chapter 4  Dynamics I, Two-dimensional motion
5. Chapter 5  Forces and Free Body Diagrams
6. Chapter 6  Force and Newton’s 1st and 2nd Laws
7. Chapter 7  Newton’s 3rd Law

Exam will reflect most key points (but not all)
25-30% of the exam will be more conceptual
70-75% of the exam is problem solving
Example with pulley

A mass $M$ is held in place by a force $F$. Find the tension in each segment of the massless ropes and the magnitude of $F$.

- Assume the pulleys are massless and frictionless.
- The action of a massless frictionless pulley is to change the direction of a tension.
- This is an example of static equilibrium.

\[ T_5 = Mg = T_2 + T_3 = 2F \quad T = Mg/2 \]
Another example with a pulley

Three blocks are connected on the table as shown. The table is frictionless & the masses are $m_1 = 4.0 \text{ kg}$, $m_2 = 1.0 \text{ kg}$ and $m_3 = 2.0 \text{ kg}$.

(A) 3 Free Body Diagrams

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Another example with a pulley

Three blocks are connected on the table as shown. The table is frictionless & the masses are $m_1 = 4.0 \text{ kg}$, $m_2 = 1.0 \text{ kg}$ and $m_3 = 2.0 \text{ kg}$.

(1) $m_1 a_{1y} = -m_1 g + T_1$
(2) $m_2 a_{2x} = -T_1 + T_3$
(3) $m_3 a_{3y} = -m_3 g + T_3$

Let $a = a_{1y} = a_{12y} = a_{3y}$ or $m_3 a = m_3 g - T_3$

Add (1) & (2) \( (m_1 + m_2)a = -m_1 g + T_3 \)

Now add (3)

\[
(m_1 + m_2 + m_3)a = -m_1 g + m_3 g
\]

\[
a = (-m_1 g + m_3 g) / (m_1 + m_2 + m_3) = -20 / 7 \text{ m/s}^2
\]
Problem recast as 1D motion

Three blocks are connected on the table as shown. The center table has a coefficient of kinetic friction of $\mu_k = 0.40$, the masses are $m_1 = 4.0$ kg, $m_2 = 1.0$ kg and $m_3 = 2.0$ kg.

$m_1g > m_3g$ and $m_1g > (\mu_k m_2g + m_3g)$

and friction opposes motion (starting with $v = 0$)

so $f_i$ is to the right and $a$ is to the left (negative)

Another example with a pulley

Three blocks are connected on the table as shown. The table has a coefficient of kinetic friction of $\mu_k = 0.40$, the masses are $m_1 = 4.0$ kg, $m_2 = 1.0$ kg and $m_3 = 2.0$ kg.

(A) FBD (except for friction)

(B) So what about friction?
Problem recast as 1D motion

Three blocks are connected on the table as shown. The center table has a coefficient of kinetic friction of $\mu_k=0.40$, the masses are $m_1 = 4.0 \text{ kg}$, $m_2 = 1.0 \text{ kg}$ and $m_3 = 2.0 \text{ kg}$.

![Diagram of three blocks connected on a table with forces labeled $m_1 g$, $m_2 g$, and $m_3 g$.]

**x-dir**: 1. \[ \sum F_x = m_2a = \mu_k m_2g - T_1 + T_3 \\
\]
\[ m_3a = m_3g - T_3 \]
\[ m_1a = -m_1g + T_1 \]

Add all three: \((m_1 + m_2 + m_3) a = \mu_k m_2g + m_3g - m_1g\)

Analyzing motion plots

1. The graph is a plot of velocity versus time for an object. Which of the following statements is correct?
   A The acceleration of the object is zero.
   B The acceleration of the object is constant.
   C The acceleration of the object is positive and increasing in magnitude.
   D The acceleration of the object is negative and decreasing in magnitude.
   E The acceleration of the object is positive and decreasing in magnitude.

![Graph of velocity versus time with a curve increasing.]
Chapter 2

**General Principles**

Kinematics describes motion in terms of position, velocity, and acceleration. General kinematic relationships are given mathematically by:

- Instantaneous velocity: \( v_i = \frac{ds}{dt} = \text{slope of position graph} \)
- Instantaneous acceleration: \( a_i = \frac{dv_i}{dt} = \text{slope of velocity graph} \)

**Motion with constant acceleration** is uniformly accelerated motion. The kinematic equations are:

\[
\begin{align*}
    v_f &= v_i + a_i \Delta t \\
    s_f &= s_i + v_i \Delta t + \frac{1}{2} a_i (\Delta t)^2 \\
    v_f^2 &= v_i^2 + 2a_i s_i
\end{align*}
\]

Uniform motion is motion with constant velocity and zero acceleration: \( s_f = s_i + v_i \Delta t \)

Important Concepts

Position, velocity, and acceleration are related graphically:
- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- \( v \) is a maximum or minimum at a turning point, and \( v = 0 \).

Also average speed and average velocity
Chapter 3

### Important Concepts

A vector is a quantity described by both a magnitude and a direction.

The vector describes the situation at this point.

The length or magnitude is denoted $A$. Magnitude is a scalar.

### Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors $i$ and $j$ define the directions of the $x$- and $y$-axes.

The components $A_x$ and $A_y$ are the magnitudes of the component vectors $A_x$ and $A_y$, and a plus or minus sign to show whether the component vector points toward the positive end or the negative end of the axis.

### Using Vectors

**Components**

The component vectors are parallel to the $x$- and $y$-axes:

$\vec{A} = A_x \hat{i} + A_y \hat{j}$

In the figure at the right, for example:

- $A_x = A \cos \theta$
- $A_y = A \sin \theta$
- $\theta = \tan^{-1}(A_y/A_x)$

- Minus signs need to be included if the vector points down or left.

### Working Graphically

- **Addition**: $\vec{A} + \vec{B}$
- **Negative**: $\vec{B}$
- **Subtraction**: $\vec{A} - \vec{B}$
- **Multiplication**: $c \vec{A}$

### Working Algebraically

Vector calculations are done component by component:

- $\vec{C} = 2\vec{A} + \vec{B}$ means $C_x = 2A_x + B_x$, $C_y = 2A_y + B_y$

The magnitude of $\vec{C}$ is then $C = \sqrt{C_x^2 + C_y^2}$ and its direction is found using $\tan^{-1}$.
Chapter 4

General Principles

The instantaneous velocity
\[ \vec{v} = \frac{d\vec{x}}{dt} \]
is a vector tangent to the trajectory.
The instantaneous acceleration is
\[ \vec{a} = \frac{d\vec{v}}{dt} \]
\(\vec{a}_n\), the component of \(\vec{a}\) parallel to \(\vec{v}\), is responsible for change of speed. \(\vec{a}_n\), the component of \(\vec{a}\) perpendicular to \(\vec{v}\), is responsible for change of direction.

Relative motion

Inertial reference frames move relative to each other with constant velocity \(\vec{V}\). Measurements of position and velocity measured in frame \(S\) are related to measurements in frame \(S'\) by the Galilean transformations:
\[ x' = x - V_x t \]
\[ y' = y - V_y t \]
\[ z' = z - V_z t \]

Important Concepts

Uniform Circular Motion

Angular velocity \(\omega = \frac{d\theta}{dt}\).
\(v\) and \(\omega\) are constant:
\[ v = r\omega \]
The centripetal acceleration points toward the center of the circle:
\[ a = \frac{v^2}{r} = r\omega^2 \]
It changes the particle's direction but not its speed.

Nonuniform Circular Motion

Angular acceleration \(\alpha = \frac{d\omega}{dt}\).
The radial acceleration
\[ a_r = \frac{v^2}{r} = \frac{r\omega^2}{r} = \omega^2 \]
changes the particle's direction. The tangential component
\[ a_t = \alpha r \]
changes the particle's speed.

Applications

Kinematics in two dimensions

If \(\vec{a}\) is constant, then the x- and y-components of motion are independent of each other:
\[ x(t) = x_0 + v_x t + \frac{1}{2} a_x t^2 \]
\[ y(t) = y_0 + v_y t + \frac{1}{2} a_y t^2 \]
\[ v_x = v_{x0} + a_x t \]
\[ v_y = v_{y0} + a_y t \]

Projectile motion occurs if the object moves under the influence of only gravity. The motion is a parabola.

- Uniform motion in the horizontal direction with \(v_{x0} = v_0 \cos \theta\).
- Free-fall motion in the vertical direction with \(v_{y0} = v_0 \sin \theta\).
- The x and y kinematic equations have the same value for \(\Delta t\).

Circular motion kinematics

Period: \(T = \frac{2\pi}{\omega} = \frac{2\pi r}{v} = \frac{\Delta t}{\Delta \theta} \)
Angular position: \(\theta = \frac{x}{r} \)
\[ \omega = \frac{\Delta \theta}{\Delta t} = \frac{\Delta x}{r \Delta t} \]
Angle, angular velocity, and angular acceleration are related graphically.
- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.
Chapter 5

General Principles

Newton's First Law
An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.

The first law tells us that no “cause” is needed for motion. Uniform motion is the “natural state” of an object.

Important Concepts

Acceleration is the link to kinematics. From \( a = \frac{\text{d}v}{\text{d}t} \), find \( \vec{a} \).
From \( v \), find \( s \) and \( a \).
\( \vec{a} = 0 \) is the condition for equilibrium.
Static equilibrium if \( \vec{v} = 0 \).
Dynamic equilibrium if \( \vec{v} = \text{constant} \).
Equilibrium occurs if and only if \( \vec{F}_{\text{net}} = 0 \).

Mass is the resistance of an object to acceleration. It is an intrinsic property of an object.

Force is a push or a pull on an object.
\( \bullet \) Force is a vector, with a magnitude and a direction.
\( \bullet \) Force requires an agent.
\( \bullet \) Force is either a contact force or a long-range force.

Equation of Motion

\[ \vec{F}_{\text{net}} = m \vec{a} \]

where \( \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \) is the vector sum of all the individual forces acting on the object.

The second law tells us that a net force causes an object to accelerate. This is the connection between force and motion that we are seeking.

Chapter 5 & 6

Key Skills

Identifying Forces
Forces are identified by locating the points where other objects touch the object of interest. These are points where contact forces are exerted. In addition, objects with mass feel a long-range gravitational force.

Free-Body Diagrams
A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.

General Strategy
All examples in this chapter follow a four-part strategy. You’ll become a better problem solver if you adhere to it as you do the homework problems. The “Dynamics Workbook” in the Student Workbook will help you structure your work in this way.

Equilibrium Problems
Object at rest or moving with constant velocity.

MODEL: Make simplifying assumptions.
VISUALIZE:
\( \bullet \) Translate words into symbols.
\( \bullet \) Identify forces.
\( \bullet \) Draw a free-body diagram.
SOLVE:
Use Newton’s first law:
\[ \vec{F}_{\text{net}} = \sum \vec{F} = 0 \]
“Read” the vectors from the free-body diagram.
ASSESS:
Is the result reasonable?

Dynamics Problems
Object accelerating.

MODEL: Make simplifying assumptions.
VISUALIZE:
\( \bullet \) Translate words into symbols.
\( \bullet \) Draw a motion diagram.
\( \bullet \) Identify forces.
\( \bullet \) Draw a free-body diagram.
SOLVE:
Use Newton’s second law:
\[ \vec{F}_{\text{net}} = \sum \vec{F} = ma \]
“Read” the vectors from the free-body diagram.
Use kinematics to find velocities and positions.
ASSESS:
Is the result reasonable?
Chapter 6

Important Concepts

<table>
<thead>
<tr>
<th>Specific information about three important forces:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity: $\vec{F}_g = (mg, \text{downward})$</td>
</tr>
<tr>
<td>Friction: $\vec{f}_f = (0 \text{ to } \mu \vec{n}, \text{direction as necessary to prevent motion})$</td>
</tr>
<tr>
<td>Drag: $\vec{D} \propto (1/2 \rho v^2, \text{direction opposite the motion})$</td>
</tr>
</tbody>
</table>

Note: Drag in air is proportional to $v^2$

Newton’s laws are vector expressions. You must write them out by components:

$$(F_{net})_x = \sum F_x = ma_x \text{ or } 0$$

$$(F_{net})_y = \sum F_y = ma_y \text{ or } 0$$

Chapter 7

General Principles

Newton’s Third Law

Every force occurs as one member of an action/reaction pair of forces. The two members of an action/reaction pair:

- Act on two different objects.
- Are equal in magnitude but opposite in direction:

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

Applications

Acceleration constraints

Objects that are constrained to move together must have accelerations of equal magnitude: $a_A = a_B$

Strings and pulleys

The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:

- Massless, or
- In equilibrium

Objects connected by massless strings passing over massless, frictionless pulleys act as if they interact via an action/reaction pair of forces.
Conceptual Problem

The pictures below depict cannonballs of identical mass which are launched upwards and forward. The cannonballs are launched at various angles above the horizontal, and with various velocities, but all have the same vertical component of velocity.

![Cannonballs](image)

Which of the following correctly ranks the time the balls are in the air, from shortest to longest?

a) I, II, III
b) III, II, I
c) I, III, II
d) All the balls are in the air for the same amount of time.

Graphing problem

The figure shows a plot of velocity vs. time for an object moving along the x-axis. Which of the following statements is true?

(A) The average acceleration over the 11.0 second interval is -0.36 m/s²

(B) The instantaneous acceleration at t = 5.0 s is -4.0 m/s²

(C) Both A and B are correct.

(D) Neither A nor B are correct.

Note: $\Delta x \neq \frac{1}{2} a_{\text{avg}} \Delta t^2$
Conceptual Problem

A block is pushed up a $20^\circ$ ramp by a 15 N force which may be applied either horizontally (P1) or parallel to the ramp (P2). How does the magnitude of the normal force N depend on the direction of P?

(A) N will be smaller if P is horizontal than if it is parallel the ramp.
(B) N will be larger if P is horizontal than if it is parallel to the ramp.
(C) N will be the same in both cases.
(D) The answer will depend on the coefficient of friction.

Conceptual Problem

A cart on a roller-coaster rolls down the track shown below. As the cart rolls beyond the point shown, what happens to its speed and acceleration in the direction of motion?

A. Both decrease.
B. The speed decreases, but the acceleration increases.
C. Both remain constant.
D. The speed increases, but acceleration decreases.
E. Both increase.
F. Other
Sample Problem

A 200 kg wood crate sits in the back of a truck. The coefficients of friction between the crate and the truck are $\mu_s = 0.9$ and $\mu_k = 0.5$.

The truck starts moving up a 20° slope. What is the maximum acceleration the truck can have without the crate slipping out the back?

Solving:

1. Visualize the problem, Draw a picture if necessary
2. Identify the system and make a Free Body Diagram
3. Choose an appropriate coordinate system
4. Apply Newton’s Laws with conditional constraints (friction)
5. Solve

Sample Problem

A physics student on Planet Exidor throws a ball that follows the parabolic trajectory shown. The ball’s position is shown at one-second intervals until $t = 3$ s. At $t = 1$ s, the ball’s velocity is $v = (2\hat{i} + 2\hat{j})$ m/s.

a. Determine the ball’s velocity at $t = 0$ s, 2 s, and 3 s.

b. What is the value of $g$ on Planet Exidor?
Sample Problem

You have been hired to measure the coefficients of friction for the newly discovered substance jelloium. Today you will measure the coefficient of kinetic friction for jelloium sliding on steel. To do so, you pull a 200 g chunk of jelloium across a horizontal steel table with a constant string tension of 1.00 N. A motion detector records the motion and displays the graph shown.

What is the value of $\mu_k$ for jelloium on steel?

\[
\Sigma F_x = m \ddot{a} = F - f = F - \mu_k N = F - \mu_k mg
\]

\[
\Sigma F_y = 0 = N - mg
\]

\[
\mu_k = \frac{(F - ma)}{mg} \quad \& \quad x = \frac{1}{2} a t^2 \quad 0.80 \text{ m} = \frac{1}{2} a 4 \text{ s}^2
\]

\[
a = 0.40 \text{ m/s}^2
\]

\[
\mu_k = \frac{(1.00 - 0.20 \cdot 0.40)}{(0.20 \cdot 10.0)} = 0.46
\]
Exercise: Newton’s 2nd Law

A force of 2 Newtons acts on a cart that is initially at rest on an air track with no air and pushed for 1 second. Because there is friction (no air), the cart stops immediately after I finish pushing. It has traveled a distance, $D$.

Next, the force of 2 Newtons acts again but is applied for 2 seconds. The new distance the cart moves relative to $D$ is:

- A. 8 x as far
- B. 4 x as far
- C. 2 x as far
- D. 1/4 x as far

Exercise: Solution

We know that under constant acceleration, $\Delta x = a (\Delta t)^2/2$ (when $v_0=0$)

Here $\Delta t_2=2\Delta t_1$, $F_2 = F_1 \Rightarrow a_2 = a_1$

$$\frac{\Delta x_2}{\Delta x_1} = \frac{\frac{1}{2}a(\Delta t_2)^2}{\frac{1}{2}a(\Delta t_1)^2} = \frac{(2\Delta t_1)^2}{\Delta t_1^2} = 4$$

(B) 4 x as long
Another question to ponder

How high will it go?

One day you are sitting somewhat pensively in an airplane seat and notice, looking out the window, one of the jet engines running at full throttle. From the pitch of the engine you estimate that the turbine is rotating at 3000 rpm and, give or take, the turbine blade has a radius of 1.00 m. If the tip of the blade were to suddenly break off (it occasionally does happen with negative consequences) and fly directly upwards, then how high would it go (assuming no air resistance and ignoring the fact that it would have to penetrate the metal cowling of the engine.)

\[ v = \omega r = 3000 \text{ rpm} = (3000 \times 2\pi / 60) \text{ rad/s} = 314 \text{ rad/s} \]
\[ v = 1.00 \text{ m} \]
\[ v_0 = \omega r = 314 \text{ m/s (~650 mph!)} \]
\[ h = h_0 + v_0 t - \frac{1}{2} g t^2 \]
\[ v_h = 0 = v_0 - g t \quad t = v_0 / g \]

So
\[ h = v_0 t - \frac{1}{2} g t^2 = \frac{1}{2} v_0^2 / g = 0.5 \times 314^2 / 9.8 = 5 \text{ km} \]
or ~ 3 miles
Sample exam problem

An object is at first travelling due north, turns and finally heads due west while increasing its speed. The average acceleration for this maneuver is pointed

A directly west.
B somewhere between west and northwest.
C somewhere between west and southwest.
D somewhere between northwest and north.
E somewhere between southwest and south.
F None of these are correct

Sample exam problem

An object is at first travelling due north, turns and finally heads due west while increasing its speed. The average acceleration for this maneuver is pointed

\[ \mathbf{a} = \frac{(\mathbf{v}_f - \mathbf{v}_i)}{\Delta t} \]

A directly west.
B somewhere between west and northwest.
\[ \text{C somewhere between west and southwest.} \]
D somewhere between northwest and north.
E somewhere between southwest and south.
F None of these are correct
Sample exam problem

A small block moves along a frictionless incline which is $45^\circ$ from horizontal. Gravity acts down at 10 m/s$^2$. There is a massless cord pulling on the block. The cord runs parallel to the incline over a pulley and then straight down. There is tension, $T_1$, in the cord which accelerates the block at 2.0 m/s$^2$ up the incline. The pulley is suspended with a second cord with tension, $T_2$.

A. What is the tension magnitude, $T_1$, in the 1st cord?
B. What is the tension magnitude, $T_2$, in the 2nd cord?
(Assume $T_1 = 50$. N if you don’t have an answer to part A.)

Sample exam problem

$a = 2.0$ m/s$^2$ up the incline.

What is the tension magnitude, $T_1$, in the 1st cord?

Use a FBD!

Along the block surface

$\sum F_x = m a_x = -mg \sin \theta + T$

$T = 5 \times 2 \text{ N} + 5 \times 10 \times 0.7071 \text{ N}$

$= (10 + 35) \text{ N} = 45 \text{ N}$
Sample exam problem

\( a = 0.0 \, \text{m/s}^2 \) at the pulley.

What is the tension magnitude, \( T_2 \), in the 2\textsuperscript{nd} cord?

Use a FBD!

Conceptual Problem

A person initially at point \( P \) in the illustration stays there a moment and then moves along the axis to \( Q \) and stays there a moment. She then runs quickly to \( R \), stays there a moment, and then strolls slowly back to \( P \). Which of the position vs. time graphs below correctly represents this motion?
Sample exam problem

You have a 2.0 kg block that moves on a linear path on a horizontal surface. The coefficient of kinetic friction between the block and the path is $\mu_k$. Attached to the block is a horizontally mounted massless string as shown in the figure below. The block includes an accelerometer which records acceleration vs. time. As you increase the tension in the rope the block experiences an increasingly positive acceleration. At some point in time the rope snaps and then the block slides to a stop (at a time of 10 seconds). Gravity, with $g = 10 \text{ m/s}^2$, acts downward.

Sample exam problem

A. At what time does the string break and, in one sentence, explain your reasoning?

B. What speed did the block have when the string broke?

C. What is the value of $\mu_k$?

D. Using $\mu_k$ above (or a value of 0.25 if you don’t have one), what was the tension in the string at $t = 2 \text{ seconds}$?
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Sample exam problem

B. What speed did the block have when the string broke?
Don’t know initial $v$ ($t=0$) so can’t integrate area at $t < 4$ sec.
$v_f = 0 \text{ m/s}$ and from $t = 4$ to 10 sec (6 second) $a = -2 \text{ m/s}^2$
$0 = v_i + a t = v_i - 2 \times 6 \text{ m/s} \quad v_i = 12 \text{ m/s}$

![Diagram](image1.png)

Sample exam problem

C. What is the value of $\mu_k$?
Use a FBD!

$\sum F_x = m \ a_x = -f_k = -\mu_k \ N$
$\sum F_y = 0 = mg - N \quad N = mg$
So $m \ a_x = -f_k = -\mu_k \ mg \quad \mu_k = -\ a_x / g = -(-2)/10 = 0.20$

![Diagram](image2.png)
Sample exam problem

D. What was the tension in the string at \( t = 2 \) seconds?

Again a FBD!

\[
\Sigma F_x = m a_x = -f_k + T
\]

\[
\Sigma F_y = 0 = mg - N \quad N = mg
\]

\[
T = f_k + m a_x = (0.20 \times 2 \times 10 + 2 \times 3) \quad N = 10 \text{ N}
\]

Recap

Exam Thursday, Oct. 6\(^{th}\) from 7:15-8:45 PM Chapters 1-6, 7 One 8½ X 11 hand written note sheet and a calculator (for trig.)