Physics 202, Lecture 24

Exam 3 Review

- **Logistics**: (November 30, 5:30-7PM; details see email)
- **Topics**: Inductance, AC Circuits, Waves, EM waves
  (Chapters 28.6-28.9, 29, 30, and materials on waves)

- Inductance:
  self inductance, mutual inductance, magnetic energy, RL circuit, ...
- AC Circuits:
  RC circuits, LC circuits, RLC circuits, impedance, phasers, ...
- Waves:
  transverse/longitudinal waves, wave function, sinusoidal waves, ...
- EM Waves:
  Maxwell’s equations, energy carried by EM waves, Poynting vector, EM wave spectrum, antennas, ...


Disclaimer

- This review is a supplement to your own preparation.

- Hints and exercises presented in this review are not meant to be complete.
Self Inductance

- When the current in a conducting device changes, an induced emf is produced in the opposite direction of the source current \( \Rightarrow \) self inductance

- The magnetic flux due to self inductance is proportional to \( I \):

  \[ \Phi_B = LI \]

\( \Rightarrow \) The induced emf is proportional to \( \frac{dI}{dt} \):

\[ \epsilon_L = -L \frac{dI}{dt} \]

**L:** Inductance, unit: Henry (H)
Exercise: Calculate Inductance of a Solenoid

☐ show that for an ideal solenoid:

\[ L = \frac{\mu_0 N^2 A}{l} \]

(see board)

Reminder: magnetic field inside the solenoid

\[ B = \mu_0 \frac{N}{l} I \]
Mutual Inductance

- For coupled coils:
  \[ \varepsilon_2 = -M_{12} \frac{dl_1}{dt} \]
  \[ \varepsilon_1 = -M_{21} \frac{dl_2}{dt} \]

Can prove (not here):
\[ M_{12} = M_{21} = M \]

\( \rightarrow M \): mutual inductance
(unit: also Henry)

- \( \varepsilon_2 = -M \frac{dl_1}{dt} \)
- \( \varepsilon_1 = -M \frac{dl_2}{dt} \)
Energy in an Inductor

- Energy stored in an inductor is $U = \frac{1}{2} LI^2$

- This energy is stored in the form of magnetic field: energy density $u_B = \frac{1}{2} B^2/\mu_0$ (recall: $u_E = \frac{1}{2} \varepsilon_0 E^2$)

- Compare:
  - Inductor: energy stored $U = \frac{1}{2} LI^2$
  - Capacitor: energy stored $U = \frac{1}{2} C(\Delta V)^2$
  - Resistor: no energy stored, (all energy converted to heat)
The time constant is $\tau = L/R$
1. A coil with a self-inductance of 6.5 H carries a current that is changing at a rate of 50 A/s. What is the induced EMF in the coil?
   A) 0.13 V  
   B) 7.7 V  
   C) 32 V  
   D) 65 V  
   E) 0.32 kV

   $\epsilon_L = -L \frac{dI}{dt}$

2. The self-inductance of a wire coil is a proportionality constant that relates
   A) electric field to current.  
   B) electric flux to current.  
   C) magnetic flux to current.  
   D) magnetic field to current.  
   E) voltage to current.

   $\Phi_B = LI$

3. For the two solenoids above, if $l = 50 \text{ cm}$, $N_1 = N_2 = 200$ turns and $r_1 = 5 \text{ cm}$ and $r_2 = 10 \text{ cm}$, the mutual inductance of the two solenoids

   $(4 \pi 10^{-7} \times 200/0.5) \times (\pi \times (0.05)^2) \times 200$

4. How much does the energy stored in an inductor change if the current through the inductor is doubled?
   A) it is the same  
   B) it is doubled  
   C) it is quadrupled  
   D) it is halved  
   E) it is quartered

   $U = \frac{1}{2} LI^2$

5. A solenoid is 15 cm long, has a radius of 5 cm, and has 400 turns. If it carries a current of 4 A, the magnetic energy stored in the solenoid is
   A) 84.2 mJ  
   B) 0.562 J  
   C) 3.37 J  
   D) 12.6 mJ  
   E) None of these is correct.

   $L = \frac{\mu_0 N^2 A}{l}$

6. An open switch in an RL circuit is closed at time $t = 0$, as shown. The curve that best illustrates the variation of current with time is

   A) 1  
   B) 2  
   C) 3  
   D) 4  
   E) 5
LC Circuit and Oscillation

Exercise: Find the oscillation frequency of a LC circuit

\[- \frac{q(t)}{c} - L \frac{dI(t)}{dt} = 0\]

\[q(t) / C + L \frac{d^2 q(t)}{dt^2} = 0\]

\[q(t) + \frac{1}{\omega^2} \frac{d^2 q(t)}{dt^2} = 0\]

\[\omega = \frac{1}{\sqrt{LC}}\]

\[q = Q_{\text{max}} \cos(\omega t + \phi)\]

\[I = -\omega Q_{\text{max}} \sin(\omega t + \phi)\]

Total Energy is conserved
Current And Voltages in a Series RLC Circuit

\[ \Delta v_R = (\Delta V_R)_{\text{max}} \sin(\omega t) \]
\[ \Delta v_L = (\Delta V_L)_{\text{max}} \sin(\omega t + \pi/2) \]
\[ \Delta v_C = (\Delta V_C)_{\text{max}} \sin(\omega t - \pi/2) \]

\[ \Delta V_{\text{max}} = I_{\text{max}} \sqrt{R^2 + (X_L - X_C)^2} \]

\[ \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \]

\[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \]

Quiz: Can the voltage amplitudes across each components, \((\Delta V_R)_{\text{max}}, (\Delta V_L)_{\text{max}}, (\Delta V_C)_{\text{max}}\) larger than the overall voltage amplitude \(\Delta V_{\text{max}}\)?
Impedance

- For general circuit configuration:
  \[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \text{, } \Delta V_{\text{max}} = I_{\text{max}} Z \]
  *Z*: is called Impedance.

  e.g. RLC circuit:

  \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

- In general impedance is a complex number. \( Z = Ze^{i\phi} \).
  It can be shown that impedance in series and parallel circuits follows the same rule as resistors.

  \( Z = Z_1 + Z_2 + Z_3 + \ldots \)  (in series)

  \[ \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots \]  (in parallel)

  (All impedances here are complex numbers)
7. How much does the maximum EMF produced by a generator (a rotating coil) change if its magnetic field is halved?
   A) it is the same
   B) it is increased by a factor of sixteen
   C) it is decreased by a factor of two
   D) it is increased by a factor of four
   E) it is impossible to tell given the information provided
   \[ \mathcal{E} = -N \frac{d\Phi_B}{dt} = -N \frac{d(AB\cos\theta)}{dt} = NAB \omega \sin(\omega t) \]

8. Two heaters are plugged into the same 120-V AC outlet. If one heater is rated at 1100 W, then what can be the maximum rating of the second heater in order not to exceed the 20 A trip rating on the circuit?
   A) 1100 W
   B) 1300 W
   C) 1200 W
   D) 2400 W
   E) 920 W
   \[ P_1/V + P_2/V = 20A \]

9. The figure shows the voltage and current for a device. The frequency of the voltage is
   A) 0.2 Hz
   B) 0.4 Hz
   C) 1.2 Hz
   D) 2.0 Hz
   E) 2.5 Hz
   \[ f = 1/T \]

10. If you double the frequency in the circuit shown, the capacitative reactance of the circuit
   A) increases by a factor of 2.
   B) does not change.
   C) decreases by a factor of 2.
   D) increases by a factor of 4.
   E) decreases by a factor of 4.
   \[ Z = \frac{1}{\omega C} \]

11. A 5-\( \mu \)F capacitor is charged to 30 V and is then connected in series with a 10-\( \mu \)H inductor and a 50-\( \Omega \) resistor. The current in this circuit after a long time has passed will be
   A) 0
   B) 8.83 A
   C) 15.4 A
   D) 21.2 A
   E) some value that cannot be determined from the given information.
   \[ \omega = \omega_0 = \frac{1}{\sqrt{LC}} \]

12. You have a 30-\( \mu \)H inductor and want to form a 1.0-MHz parallel, resonant circuit. You need a capacitor of
   A) approximately 0.84 nF.
   B) approximately 1.2 nF.
   C) approximately 2.1 \( \mu \)F.
   D) approximately 33 \( \mu \)F.
   E) None of these is correct.

13. As you increase the frequency in this circuit from zero,
   A) the impedance increases to a maximum and then decreases.
   B) the impedance decreases to a minimum and then increases.
   C) the impedance will decrease continuously.
   D) the impedance does not change.
   E) None of these is correct.
   \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]
Transverse and Longitudinal Waves

- If the direction of mechanic disturbance (displacement) is perpendicular to the direction of wave motion, the wave is called transverse wave.
- If the direction of mechanic disturbance (displacement) is parallel to the direction of wave motion, the wave is called longitudinal wave.

EM waves are always transverse.
Linear Wave Equation

- Linear wave equation:
  \[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]

- Sinusoidal wave:
  \[ y = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi ft + \phi \right) \]

- Wave speed

- Certain physical quantity

- General wave: superposition of sinusoidal waves
Maxwell Equations

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \]  \rightarrow \text{Gauss’s Law/ Coulomb’s Law}

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]  \rightarrow \text{Gauss’s Law of Magnetism, no magnetic charge}

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \]  \rightarrow \text{Faraday’s Law}

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]  \rightarrow \text{Ampere Maxwell Law}

Also, Lorentz force Law \rightarrow \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}

These are the foundations of the electromagnetism
Properties of EM Waves

- EM waves are transverse waves traveling at speed c:
  \[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]

- Recall: energy densities \( u_E = \frac{1}{2} \varepsilon_0 E^2 \), \( u_B = \frac{1}{2} B^2/\mu_0 \)

- For a EM wave, at any time/location,
  \[ u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 = u_B \quad \text{(using } E/B = c) \]
  \[ \rightarrow \text{In an EM wave, the energies carried by electric field and magnetic field are always the same.} \]

- Total energy stored (per unit of volume):
  \[ u = u_E + u_B = \varepsilon_0 E^2 = B^2/\mu_0 \]

- Power transmitted per unit of area is equal to \( uc \) in the direction of wave

- Averaging over time:
  \[ u_{av} = \frac{1}{2} \varepsilon_0 E_{max}^2 = \frac{1}{2} B_{max}^2/\mu_0 \quad u_{av}c = I \text{ (intensity)} \]
The Poynting Vector

- The rate of flow of energy in an electromagnetic wave is described by a vector, \( \mathbf{S} \), called the Poynting vector.
- The Poynting vector is defined as:
  \[
  \mathbf{S} \equiv \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}
  \]
  \[ I = S_{av} \]
- Its direction is the direction of propagation.
- This is time dependent:
  - Its magnitude varies in time.
  - Its magnitude reaches a maximum at the same instant as \( \mathbf{E} \) and \( \mathbf{B} \).
Antennas

- Antennas are essentially an arrangement of conductors for transmitting and receiving radio waves.
- Parameters: gain, impedance, frequency, orientation, polarization, etc.

![Diagram of half-wave antenna](image)

**λ/4**

**maximum strength**

**half-wave antenna**

**λ/4**

**mocristrip**

**loop**

**log-periodic**
14. A parallel-plate capacitor has closely spaced circular plates of radius $R = 3.00$ cm. Charge is flowing onto the positive plate at the rate $I = dQ/dt = 3.65$ A. The magnetic field at a distance $r = 1.50$ cm from the axis of the plates is approximately
A) 135 mT
B) 256 μT
C) 1.35 μT
D) 457 mT
E) 88.3 μT

18. You are using an antenna consisting of a single loop of wire of radius 15.0 cm to detect electromagnetic waves for which $E_{\text{rms}} = 0.200$ V/m. If the wave frequency is 600 Hz, the rms value of the emf induced in the loop is approximately

\[ E = c B \]

\[ I = \text{Power/area} \]

\[ E_{\text{rms}}^2 = \frac{1}{2} E_{\text{max}}^2 \]

\[ u_{\text{av}} = \frac{1}{2} \varepsilon_0 E_{\text{max}}^2 = \frac{1}{2} B_{\text{max}}^2/\mu_0, \quad u_{\text{av}} c = I \]