Physics 202, Lecture 21

Today’s Topics
- Electromagnetic Waves (EM Waves)
- The Hertz Experiment
- Review of the Laws of Electro-Magnetism
- Maxwell’s equations
- Propagation of $E$ and $B$
- The Linear Wave Equation

Demo: Hertz Experiment
In 1887, Heinrich Hertz first demonstrated that EM fields can transmit over space.

Hertz Experiment: Conceptual Schematic

Review: (1) Gauss’s Law / Coulomb’s Law
- The relation between the electric flux through a closed surface and the net charge $q$ enclosed within that surface is given by the Gauss’s Law

$$ \oint E \cdot d\vec{A} = \frac{q}{\varepsilon_0} $$

(2) Gauss’s Law for Magnetism
- The Gauss’s Law for the electric flux is a reflection of the existence of electric charge. In nature we have not found the equivalent, a magnetic charge, or monopole.
- We can express this result differently: if any closed surface as many lines enter the enclosed volume as they leave it

$$ \oint B \cdot d\vec{A} = 0 $$

Magnetic Monopole
- Paul Dirac argued that the existence of magnetic monopole may explain the observed quantization of electric charges.
- Magnetic monopole has not been detected.
- “Although there have been tantalizing events recorded, in particular the event recorded by Blas Cabrera on the night of February 14, 1982 (thus, sometimes referred to as the “Valentine’s Day Monopole”), there has never been reproducible evidence for the existence of magnetic monopoles.” -- Wikipedia
Review: (3) Faraday’s Law
- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit” or closed path.
  \[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]
- Since \( \mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} \)
- Then \( \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \)

Review: (4) Ampere’s Law
- A magnetic field is produced by an electric current as given by the Ampere’s Law
  \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \]
- A changing electric field will also produce a magnetic field.
Finally:
  \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \epsilon_0 \frac{d\Phi_E}{dt} \]

Maxwell Equations
\[
\begin{align*}
\oint \mathbf{E} \cdot dA &= \frac{Q}{\varepsilon_0} \quad \rightarrow \text{Gauss’s Law/Coulomb’s Law} \\
\oint \mathbf{B} \cdot dA &= 0 \quad \rightarrow \text{Gauss’s Law of Magnetism, no magnetic charge} \\
\oint \mathbf{E} \cdot d\mathbf{l} &= -\frac{d\Phi_B}{dt} \quad \rightarrow \text{Faraday’s Law} \\
\oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I + \epsilon_0 \frac{d\Phi_E}{dt} \quad \rightarrow \text{Ampere Maxwell Law}
\end{align*}
\]

Also, Lorentz force law:
\[ \mathbf{F} = q \mathbf{E} + q \mathbf{v} \times \mathbf{B} \]

These are the foundations of the electromagnetism.

Linear Wave Equation
- Linear wave equation
- Sinusoidal wave

General wave: superposition of sinusoidal waves

EM Fields in Space
- Maxwell equations when there is no charge and current:
  \[
  \begin{align*}
  \oint \mathbf{E} \cdot dA &= 0 \\
  \oint \mathbf{B} \cdot dA &= 0 \\
  \oint \mathbf{E} \cdot d\mathbf{l} &= -\frac{d\Phi_B}{dt} \\
  \oint \mathbf{B} \cdot d\mathbf{l} &= \mu_0 I + \epsilon_0 \frac{d\Phi_E}{dt}
  \end{align*}
  \]

differential forms:
- (single polarization):
  \[
  \frac{\partial E_x}{\partial x} = -\frac{\partial B_z}{\partial t}, \quad \frac{\partial B_z}{\partial t} = -\mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}
  \]

Electromagnetic Waves
- EM wave equations:
  \[
  \frac{\partial^2 E_x}{\partial t^2} = \mu_0 \frac{\partial^2 B_y}{\partial z^2}, \quad \frac{\partial^2 B_y}{\partial t^2} = \mu_0 \frac{\partial^2 E_x}{\partial z^2}
  \]
- Plane wave solutions:
  \[ E = E_{\text{max}} \cos(kx - \omega t + \phi), \quad B = B_{\text{max}} \cos(kx - \omega t + \phi) \]
- Properties:
  - No medium is necessary.
  - \( E \) and \( B \) are normal to each other.
  - \( E \) and \( B \) are in phase.
  - Direction of wave is normal to both \( E \) and \( B \).
  - Speed of EM wave:
    \[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s} \]
  - \( E/B \) or \( E_{\text{max}}/B_{\text{max}} c \)
  - Transverse wave: two polarizations possible.
The EM Wave

Two polarizations possible (showing one)