Physics 202, Lecture 6

Today’s Topics

- Electric Potential (Ch. 23)
- Electric Potential For Various Charge Distributions
  - Point charges
  - Continuous charges
    e.g. Uniform Ring, Sphere, Shell
- Equipotential surfaces, Electrostatic equilibrium of conductors

- Expected from preview:
  - $E \leftrightarrow V$ relationship, equipotential lines, electrostatic equilibrium of conductors…
  - Electric potential for a charge distribution…
Review: Electric Potential Difference

- Electric Potential Energy: \( q \) In a Generic E. Field

\[
\Delta U = U_B - U_A = -q \int_A^B \mathbf{E} \cdot ds = q \Delta V
\]

- Electric Potential Difference

\[
\Delta V \equiv \frac{\Delta U}{q} = -\int_A^B \mathbf{E} \cdot ds = V_B - V_A
\]
Electric Potential Difference
For Single Point Charge (Lecture 5 Review)

- \( V_B - V_A = k_e \left( \frac{q}{r_B} - \frac{q}{r_A} \right) \)

Take \( V_\infty = 0 \) →

\[ V = k_e \frac{q}{r} \]
Electric Potential For Continuous Charge Distribution

- For finite charge distribution, it is common to set $V=0$ at infinite.

\[ V = k_e \int \frac{dq}{r} \]

- If the charge distribution is known, $V$ can be calculated simply by scalar integral.

\[ dV = k_e \frac{dq}{r} \quad (V=0 \ @ \ \infty) \]
For a uniformly charged ring, show that the potential along the central axis is

$$V = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$

**Solution**

$$V = \int \frac{k_e dq}{r}$$

$$= \int \frac{k_e dq}{\sqrt{x^2 + a^2}}$$

$$= \frac{k_e}{\sqrt{x^2 + a^2}} \int dq$$

$$= Q$$
Uniformly Charged Ring: Electric Field

- Find the electric field along the central axis.

**Approach 1: Superposition.**

- \( dE_x = dE \cos \theta = \frac{k_e dq x}{r^2 r} \)
- \( E_x = \int dE_x = \frac{k_e x Q}{r^3} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}} \)
- \( E_\perp = 0 \) due to symmetry

**Approach 2: derivative of potential**

- \( E_x = - \frac{\partial V}{\partial x} = \frac{k_e x Q}{(x^2 + a^2)^{3/2}} \)
Example: Uniformly Charged Spherical Shell

- For uniformly charged spherical shell.

Again, use:

\[ \Delta V = -\int_{A}^{B} \mathbf{E} \cdot d\mathbf{s} = V_B - V_A \]

Tip:
V is the same inside E=0 region

\[ E = 0 \quad r < R \]

\[ E_r = \frac{k_e Q}{r^2} \quad r > R \]
Example: Uniformly Charged Sphere

Show that for a uniformly charged sphere, the electric potential is:

\[ V_0 = \frac{3k_eQ}{2R} \]

\[ V_D = \frac{k_eQ}{2R} \left( 3 - \frac{r^2}{R^2} \right) \]

\[ V_B = \frac{k_eQ}{r} \]

\[ \Delta V = -\int_A^B \mathbf{E} \cdot ds = V_B - V_A \]

Since from Gauss’s law:

\[ E_r = \frac{k_eQ}{r^2} \quad r > R \]

\[ E_r = \frac{k_eQr}{R^3} \quad r < R \]
Summary: Methods for Obtaining $E$, $V$

Coulomb’s Law:  
(lecture 2)  
$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$

Gauss’s Law:  
(lecture 3, 4)  
$$\int \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

Potential:  
(lecture 5, today)  
$$V = k \int \frac{dq}{r} \quad \rightarrow \quad \vec{E} = -\nabla V$$  

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

Know all methods (and when to apply which)!
Conductors in Electrostatic Equilibrium

We have learned that:

- Electric field inside conductor is zero (also zero inside any empty cavity within the conductor)
- All net charges reside on the surface
- Electric field on surface of conductor always normal to the surface, magnitude $\sigma/\varepsilon_0$
- Sharper edge $\rightarrow$ larger field.

Since $E=0$ inside conductor,

- Potential is the same throughout the conductor: Equipotential
Charge Distribution on Conductors: Field lines and Equipotential Surfaces
Review: Charge Distribution On Conductors (I)

\[ |E| \text{ higher at smaller radius of curvature (more charge density)} \]

\[ |E| \text{ lower (less charge density)} \]
High Voltage Electrostatic Generator: Van de Graaff Generator

up to $3 \times 10^6$V
(~50KV in today’s demo)

$10^4$V
Two conductors are connected by a wire. How do the potentials at the conductor surfaces compare?

a) $V_A > V_B$  
b) $V_A = V_B$  
c) $V_A < V_B$

What happens to the charge on conductor A after it is connected to conductor B?

a) $Q_A$ increases  
b) $Q_A$ decreases  
c) $Q_A$ doesn’t change