Today’s Topics

- Electric Potential (Ch. 23)
  - Electric Potential Energy and Electric Potential
  - Electric Potential and Electric Field
Electric Potential Energy and Electric Potential

Review: Conservation of Energy (particle)

**Kinetic Energy (K)**. **Potential Energy U**: for conservative forces (can be defined since work done by F is path-independent)

\[ K = \frac{1}{2}mv^2 \quad U(x, y, z) \]

If only conservative forces present in system, conservation of mechanical energy: \[ K + U = \text{constant} \]

Conservative forces:
- Springs: elastic potential energy \[ U = k_{spring}x^2/2 \]
- Gravity: gravitational potential energy
- Electrostatic: electric potential energy (analogy with gravity)

Nonconservative forces
- Friction, viscous damping (terminal velocity)
Electric Potential Energy

Given two positive charges \( q \) and \( q_0 \): initially very far apart, choose \( U_i = 0 \)

To push particles together requires work (they want to repel). Final potential energy will increase! \( \Delta U = U_f - U_i = \Delta W \)

If \( q \) fixed, what is work needed to move \( q_0 \) a distance \( r \) from \( q \)?

\[
\Delta W = \int_{\infty}^{r} \vec{F}_{us} \cdot d\vec{l} = -\int_{r'}^{\infty} \vec{F}_{us} \cdot d\vec{l} = -\int \frac{kqq_0}{r'^2} dr' = \frac{kqq_0}{r}
\]

Note: if \( q \) negative, final potential energy negative
Particles will move to minimize their final potential energy!
Electric Potential Energy

Electric potential energy between two point charges:

\[ U(r) = \frac{kq_0q}{r} \]

- U is a scalar quantity, can be + or -
- convenient choice: \( U = 0 \) at \( r = \infty \)
- SI unit: Joule (J)

Electric potential energy for system of multiple charges: sum over pairs:

\[ U(r) = \sum_{i<j} \sum_j \frac{kq_iq_j}{r_{ij}} \]

This is the work required to assemble charges.
Example: Three Charge system

- What is the work required to assemble the three charge system as shown? \((q_1=q_2=q_3=Q)\)

Answer: \(k_e 3Q^2/a\) (see board)

- Quiz: What if \(q_1=q_2=Q\) but \(q_3=-Q\) ?

Answer: \(-k_e Q^2/a\)
Electric Potential

Charge \( q_0 \) is subject to Coulomb force in electric field \( \mathbf{E} \). Work done by electric force:

\[
W = \int_i^f \mathbf{F} \cdot d\mathbf{l} = q_0 \int_i^f \mathbf{E} \cdot d\mathbf{l} = -\Delta U
\]

Electric Potential Difference:

\[
\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{l} = V_B - V_A
\]

Units: Volts
\((1 \text{ V} = 1 \text{ J/C})\)

Often called potential \( V \), but meaningful only as potential difference

Customary to choose reference point \( V=0 \) at \( r = \infty \)
\(\text{(OK for localized charge distribution)}\)
Electric Potential and Point Charges

For point charge $q$ shown below, what is $V_B - V_A$?

$$V_B - V_A = -\int_A^B E(r)dr = -kq \int_A^B \frac{dr}{r^2} = kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$$

independent of path b/w A and B!

Potential of point charge:

$$V(r) = \frac{kq}{r}$$

Many point charges: superposition

$$V(r) = k \sum_i \frac{q_i}{r_i}$$

Equipotentials: lines of constant potential
Electric Potential: Continuous Distributions

Two methods for calculating $V$:

1. Brute force integration (next lecture)

\[ dV = k \frac{dq}{r}, \quad V = \int dV \]

2. Obtain from Gauss’s law and definition of $V$:

\[ V(r) = -\int_{\text{ref}}^{r} \vec{E}(r) \cdot d\vec{l} \]
Obtaining the Electric Field From the Electric Potential

- Three ways to calculate the electric field
  - Coulomb’s Law
  - Gauss’s Law
  - Derive from electric potential

- Formalism

\[ \Delta V = - \int_{A}^{B} \vec{E} \cdot d\vec{s} \]

\[ dV = - \vec{E} \cdot d\vec{s} = -E_x \, dx - E_y \, dy - E_z \, dz \]

\[ E_x = - \frac{\partial V}{\partial x}, \quad E_y = - \frac{\partial V}{\partial y}, \quad E_z = - \frac{\partial V}{\partial z} \quad \text{or} \quad \vec{E} = -\nabla V \]
Compute E from V

Imagine an electric potential of the following form

\[ V(x, y, z) = 2x^2 + 8y^2z + 2z^2 \]

Units of V

\[ E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z} \quad \text{or} \quad \vec{E} = -\nabla V \]

Units of V/m

\[ E_x = -\frac{\partial V}{\partial x} = -4x \]

\[ E_y = -\frac{\partial V}{\partial y} = -16yz \]

\[ E_z = -\frac{\partial V}{\partial z} = -8y^2 - 4z \]
Exercise 1: Uniform Electric Field

In the uniform electric field shown:

1. Find potential at B,C,D,G

2. If a charge +q is placed at B, what is the potential energy $U_B$?

3. If now a –q is at B, what is $U_B$?

4. If a -q is initially at rest at G, will it move to A or B?

5. What is the kinetic energy when it reaches A?
Exercise 2: Cathode Ray Tube (CRT)

Electrons are emitted with almost zero velocity on plate C, what is the energy per electron when they reach plate A? (Do with your TA)

\[ V_A - V_C = 12000 \text{V} \]
Field lines always point towards lower electric potential.

In an electric field:

- Positive charges are always subject to a force in the direction of field lines, towards lower V.
- Negative charge is always subject a force in the opposite direction of field lines, towards higher V.