Physics 202, Lecture 20

Today’s Topics

- Power in RLC
- Resonance in RLC
- Wave Motion (Review ch. 15)
  - General Wave
    - Transverse and Longitudinal Waves
    - Wave Function
    - Wave Speed
    - Sinusoidal Waves
    - Wave and Energy Transmission

**Impedance**

- For general circuit configuration:
  \[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \]
  \[ Z = \text{impedance}. \]
  
  e.g. RLC circuit: 
  \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

- In general impedance is a complex number, \( Z = Z e^{i \phi} \).
- (the above result is specific to a RLC series circuit)
- The impedance in series and parallel circuits follows the same rule as resistors.
  
  \[ Z = Z_1 + Z_2 + Z_3 + \ldots \quad \text{(in series)} \]
  \[ \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots \quad \text{(in parallel)} \]
  
  (All impedances here can be complex numbers)

**Summary of Impedances and Phases of series circuits**

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**Comparison Between Impedance and Resistance**

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<tr>
<th>Symbol</th>
<th>Resistance</th>
<th>Impedance</th>
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<td>Z</td>
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Application: Circuits with only R

- Real
- Complex: \( Z = |Z| e^{i \phi} \)

I vs. \( \Delta V \) Relationship:

- \( \Delta V = IR \)
- \( \Delta V = HZ \)
- \( \Delta V = \text{impedance} \)

In Series:

- \( R + R_1 + R_2 + \ldots \)
- \( Z + Z_1 + Z_2 + \ldots \)

In Parallel:

- \( 1/R + 1/R_1 + 1/R_2 + \ldots \)
- \( 1/Z + 1/Z_1 + 1/Z_2 + \ldots \)

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**Resonances In Series RLC Circuit**

- The impedance of an AC circuit is a function of \( \omega \).
- e.g Series RLC:
  \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]
  \[ = \sqrt{R^2 + \left( \omega L - \frac{1}{\omega C} \right)^2} \]

  - when \( \omega = \omega_0 \)
    - \[ I = \frac{1}{\sqrt{L C}} \] (i.e. \( X_L = X_C \))
    - lowest impedance \( \rightarrow \) largest current \( \rightarrow \) resonance
    - Same as the phase of LC circuit in harmonic oscillation

- For a general AC circuit, at resonance:
  - Impedance is at lowest
  - Phase angle is zero (I is "in phase" with \( \Delta V \))
  - \( I_{\text{max}} \) is at highest
  - Power consumption is at highest

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**Power in AC Circuit**

- Power in a circuit: \( P(t) = i(t) \Delta V(t) \) true for any circuit, AC or DC
- In an AC circuit, current and voltage on any component can be written in general:
  - \( \Delta V(t) = \Delta V_{\text{max}} \sin(\omega t + \phi) \)
  - \( i(t) = I_{\text{max}} \sin(\omega t) \)
- \( P(t) = I_{\text{max}} \sin(\omega t) \times \Delta V_{\text{max}} \sin(\omega t + \phi) \) \( \xrightarrow{\text{Power Factor}} \) \( \frac{1}{2} I_{\text{max}} \Delta V_{\text{max}} \)
- For resistor: \( \phi = 0 \)
- For inductor: \( \phi = \frac{\pi}{2} \)
- For capacitor: \( \phi = -\frac{\pi}{2} \)
- For an AC circuit at resonance:
  - \( \text{Power Factor} = 1 \)
- Widely used definitions:
  - \( \Delta V = \Delta V_{\text{max}} \)
  - \( I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} \)

**General Waves (Review of Ch. 15)**

- Wave: Propagation of a physical quantity in space over time
  - \( q = q(x, t) \)
- Examples of waves:
  - Water wave, wave on string, sound wave, earthquake wave, electromagnetic wave, "light", quantum wave...
- Waves can be transverse or longitudinal.

**Example wave: Stretched Rope**

- It is a transverse wave
- The wave speed is determined by the tension and the linear density of the rope:
  - \( \nu = \sqrt{\frac{T}{\mu}} \)
  - \( \mu = \frac{\Delta m}{\Delta l} \)

**Seismic Waves**

- Longitudinal
- Transverse

**Electro-Magnetic Waves are Transverse**

- A changing magnetic field can cause an electric field (this electric field was the source of the Induced EMF)
- A changing electric field also cause an magnetic field (next chapter)

**Wave Function**

- Waves are described by wave functions in the form:
  - \( y(x,t) = f(x-\nu t) \)
- \( \nu \): A certain physical quantity (e.g. displacement in y direction or electric and magnetic fields)
- \( f \): Can be any form
- \( t \): time. Its coefficient \( \nu \) is the wave speed
  - \( \nu > 0 \) moving right
  - \( \nu < 0 \) moving left
- \( x \): space position
  - Coefficient arranged to be 1
Practical Technique: Identify Wave Speed in A Wave Function

- A wave function is in the form:
  \[ y(x,t) = \frac{2}{(x - 3.0t)^2 + 1} \]
- The wave speed:
  - 3.0 m/s to the right
- Illustrate wave form at \( t = 0s, 1s, 2s \)

Sinusoidal Wave: Fixed X

- A wave describe a function \( y = \sin(kx - \omega t + \phi) \) is called sinusoidal wave. (Harmonic wave)
- The wave speed: \( v = \omega / k \)
- At each fixed position x,
  - Amplitude: \( |A| \)
  - Frequency: \( f = \omega / 2\pi \)
  - Angular frequency: \( \omega \)
  - Phase constant: \( -kx - \phi \)

Sinusoidal Wave: Fixed T

- Wave: \( y = \sin(kx - \omega t + \phi) \)
- Snapshot with fixed t:
  - Amplitude: \( |A| \)
  - Wave length: \( \lambda = 2\pi / k \)
- Wave Speed \( v = \omega / k \)
  - \( v = \lambda f \), or
  - \( v = \lambda / T \)

Waves Transfer Energy

- As motion in propagating in the form of wave in a medium, energy is transmitted.
- It can be shown that the rate of energy transfer by a sinusoidal wave on a rope is:
  \[ P = \frac{1}{2} \mu A^2 \omega^2 v \]
- Note: power dependence on \( A, \omega, v \)
- We'll find that EM waves can transfer energy also!

Linear Wave Equation

- Linear wave equation
  - certain physical quantity
  \[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]
- Wave speed
- General wave: superposition of sinusoidal waves
  - EM waves will obey such a wave equation.
    - Time changing current \( \to \) Time changing Magnetic field \( \to \)
    - Time changing Electric field \( (dE/dt) \to \)
    - Time changing Magnetic Field \( (dB/dt) \to \)

Sinusoidal wave

- \( f: \) frequency
- \( \phi: \) Phase
- \( v = \lambda f \)
- \( k = 2\pi / \lambda \)
- \( \omega = 2\pi f \)