Physics 202, Lecture 14

Today’s Topics

- Sources of the Magnetic Field (Ch 27)
  - Review: The Ampere’s Law
  - Applications And Exercises of ampere’s Law
    - Straight line, Solenoid, Toroid

- Magnetism in Matter
Review: Ampere’s Law

- Ampere’s Law:
  \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \]
  for any closed path

- It applies to any closed path
- It applies to any static B field
- It is practically useful in symmetric cases

- Ampere’s Law can be derived from Biot-Savart Law
  - Key point: Derivation relies on the fact that the current has no divergence (is a continuous flow does not increase or decrease at any point). This works for current loops or infinite currents.
Solenoid

- The B field inside an ideal solenoid:
  - Ideal: Infinitely long and tightly wound

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \]

\[ B_l = \mu_0 NI \]

\[ B = \mu_0 nI \]

\( n = N/L \)

B field independent of where you are inside the solenoid
The B field inside an ideal solenoid is:

\[ B = \mu_0 n I \]

where \( n = \frac{N}{L} \)
Compare Solenoid and Bar Magnet

Loose Solenoid

Tight Solenoid

Bar Magnet
The B field inside a toroid

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \text{any closed path} \]

\[ B2\pi r = \mu_0 NI \]

\[ B = \frac{\mu_0 NI}{2\pi r} \]

b\(<r\<c: B=0 \text{ outside the Torus because the Ampere’s loop does not enclose any net current} \]
Electric dipole moment $\mathbf{p}$. Dielectric material contains electric dipoles at atomic level.

In an external field $E_0$, the dipoles line up $E_{\text{ind}}$ is always opposite to $E_0$.

$\sum \mathbf{F} = 0$

$\vec{\tau} = \mathbf{p} \times \mathbf{E}$

$U = -\mathbf{p} \cdot \mathbf{E}$

$E = E_0 / \kappa < E_0$, $C = \kappa C_0$

(dielectric constant $\kappa > 1$)
More on Magnetic Dipole Moments

Magnetic dipole moment $\mu$.

Note: $B$ produced (at the center) is always in the same direction as $\mu$.

definition of magnetic moment

Macroscopic

$\mu = I A$

Microscopic

From orbiting electrons (depends on their angular momentum) and quantum mechanical spin (special type of angular momentum)

$I = q/T = qv/2\pi r$

$\mu = I A = 1/2 qvr = (q/2m)L$

$B_{z=\infty} = \frac{\mu_0 I R^2}{2 |z|^3}$

$B = \frac{\mu_0 \mu}{2 |z|^3}$
Magnetism in Matter

- Total magnetic dipole moment $\mu$ and the magnetic field
  - If all the individual dipoles line up, the field can be very strong. Induced by applying an external field
  - Configuration like a solenoid with constant field
  - Depends on details of the material.
- Define the magnetization $M$ where $B_{\text{ind}} = \mu_0 M$

\[ \sum F = 0 \]
\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]
\[ U = -\vec{\mu} \cdot \vec{B} \]
Magnetism in Matter

- Induced field $B_{\text{ind}}$ in response to an external $B_0$: $B_{\text{ind}} = \chi B_0$
- the net field inside: $B = B_0 + B_{\text{ind}} = (1 + \chi) B_0 = (\mu_m/\mu_0)B_0 = K_m B_0$

$\mu_m$: magnetic permeability, $\chi$: magnetic susceptibility,
$K$: relative permeability

<table>
<thead>
<tr>
<th>Classification of Magnetic Matter</th>
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<tr>
<td>Type</td>
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<tr>
<td>Ferromagnetic (e.g. Fe, Co, Ni...)</td>
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<tr>
<td>Paramagnetic (e.g. Al, Ca,...)</td>
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<tr>
<td>Diamagnetic (e.g. Cu, Au,...)</td>
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<tr>
<td>Superconductor</td>
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Permanent Magnetic Moments (domains)  
Inside Ferromagnetic Material

No external $B$ field,  
permanent mag. moments exist, but oriented randomly  
$\rightarrow$ no induced $B$ field

$B_0$ applied, permanent magnetic moments line up in the direction of $B_0$  
$\rightarrow$ strong induced $B$ field

Note: Inductance with a ferromagnetic core: $B = \frac{\mu_m}{\mu_0}B_0 \gg B_0$
Meissner Effect

- Certain superconductors (type I) exhibit perfect diamagnetism in superconducting state: no magnetic field allowed inside (Meissner Effect)