Today’s Topics

- Direct Current Circuits (Ch. 25)
- Basic circuit components (ε, R, …)
- Kirchhoff’s Rules
- Circuits Analysis (For circuits of R’s and ε’s)

Exam scores posted tomorrow
# Basic Circuit Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal battery, emf</td>
<td><img src="image" alt="battery" /></td>
<td>$\Delta V = V_+ - V_- = \varepsilon$</td>
</tr>
<tr>
<td>Resistor</td>
<td><img src="image" alt="resistor" /></td>
<td>$\Delta V = -IR$</td>
</tr>
<tr>
<td>Realistic Battery</td>
<td><img src="image" alt="battery" /></td>
<td>$\Delta V = 0$ (→$R=0$, $L=0$, $C=0$)</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td><img src="image" alt="wires" /></td>
<td>$\Delta V = V_- - V_+ = -q/C$, $dq/dt = I$</td>
</tr>
<tr>
<td>Capacitor</td>
<td><img src="image" alt="capacitor" /></td>
<td>$\Delta V = -L\frac{dI}{dt}$</td>
</tr>
<tr>
<td>Inductor</td>
<td><img src="image" alt="inductor" /></td>
<td></td>
</tr>
<tr>
<td>(Ideal) Switch</td>
<td><img src="image" alt="switch" /></td>
<td>$L=0$, $C=0$, $R=0$ (on), $R=\infty$ (off)</td>
</tr>
<tr>
<td>Transformer</td>
<td><img src="image" alt="transformer" /></td>
<td>Future Topics</td>
</tr>
<tr>
<td>Diodes, Transistors,</td>
<td><img src="image" alt="diodes" /></td>
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<tr>
<td>…</td>
<td><img src="image" alt="transistors" /></td>
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</table>
Simple Circuit 1: Resistors In Series

- Equivalent resistance: \( R_{eq} = R_1 + R_2 \)
  - \( I_1 = I_2 = I \)
  - \( \Delta V = V_{R1} + V_{R2} = IR_1 + IR_2 \)
  \( \Rightarrow \Delta V = I(R_1 + R_2) \)
  i.e. \( R_{eq} = R_1 + R_2 \)

- In general for series resistors: \( R_{eq} = R_1 + R_2 + R_3 + \ldots \)
Simple Circuit 2: Resistors In Parallel

- Show $1/R_{eq} = 1/R_1 + 1/R_2$.
  - $V_{R1} = V_{R2} = \Delta V$
  - $I_1 + I_2 = I$
  - $\Rightarrow I_1 R_1 / R_1 + I_2 R_2 / R_2 = I$
  - $\Rightarrow \Delta V (1/R_1 + 1/R_2) = I$
  - $\Rightarrow \Delta V = I \cdot 1/(1/R_1 + 1/R_2)$

  i.e.

- Parallel resistors: $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$
Exercise: Equivalent Resistance of a Combined Parallel and Serial Circuit

What is the $R_{eq}$ for the combination shown?
$R_1 = R_2 = 1\, \Omega$, $R_3 = 2\, \Omega$, $R_4 = 4\, \Omega$.

1. $8\, \Omega$
2. $6\, \Omega$
3. $5\, \Omega$
4. None of above
A complicated circuit:
- May contain more than one emf
- May not be simplified as “in series” or “in parallel”
- May contain multi loops and junctions.
Kirchhoff’s Rules: Junction Rule

- Junction Rule (Charge conservation):
  The sum of currents entering any junction equals the sum of currents leaving that junction.

\[ \sum I_{\text{in}} = \sum I_{\text{out}} \]

- In practice, the classifications of “in” and “out” are determined by assigned direction for each current.
  The assignment of current directions can be arbitrary. They may not be the same as actual directions, which are not known a prior.
  - “in” : current with assigned direction towards junction
  - “out” : current with assigned direction off junction
(Very) Quick Quiz: Junction Rule

What is the junction rule for the current assignment shown?

1. \( I_1 + I_2 = I_3 \)
2. \( I_1 = I_2 + I_3 \)  \( \rightarrow \)
3. \( I_1 - I_2 = I_3 \)

Although equation 2 and 3 are equivalent, equation 3 does not follow template form \( I_{\text{in}} = I_{\text{out}} \)
Kirchhoff’s Rules: Loop Rule

- Loop Rule (Energy Conservation): PE = QV. (if no new energy is introduced):
  The sum of potential drops across components along any closed circuit loop must be zero.

\[\Sigma \Delta V = 0\]

- The potential “drop” across a component is always defined as \(V_{\text{down\_stream\_end}} - V_{\text{up\_stream\_end}}\) where, the stream direction is the loop direction.

- The exact expression of the potential drop is determined by the type of component and the assigned current direction. (See next slides)
Determine Potential Difference

\[ \Delta V = V_b - V_a = -\varepsilon \]

\[ \Delta V = V_b - V_a = +\varepsilon \]

\[ \Delta V = V_b - V_a = -IR \]

\[ \Delta V = V_b - V_a = +IR \]
Steps to Apply Kirchhoff’s Rules

1. Assign a directional current for each branch (segment) of a circuit. The assigned direction for each current can be arbitrarily chosen but, once assigned, need to be observed.

2. Set up junction rules at certain (any) junctions. (Typically you need one less than you have junctions)

3. Select a number of closed loops to apply loop rule. For each closed loop, assign a loop direction (clockwise or counter clockwise). Follow that assigned direction, find ΔV drop across each component, and apply loop rule.

4. Solve for unknowns. (if you don’t have enough equations to solve for the unknowns you probably missed some loops)

5. If a current is found to be negative, it means its actual direction is opposite to the originally chosen one. (a little thought in step 1 will lead you to more often identify the direction correctly)
Example 1: Multi-Loop

Find out \( I_1, I_2, I_3 \)

Kirchhoff’s Rules:

Junction c:
\[ I_1 + I_2 = I_3 \]

Loop abcd:
\[ \varepsilon_1 - I_1 R_1 - I_3 R_3 = 0 \]

Loop befcb:
\[ -\varepsilon_2 + I_1 R_1 - \varepsilon_1 - I_2 R_2 = 0 \]

Solving three equations:
\( I_1 = 2.0A, I_2 = -3.0A, I_3 = -1.0A, \)

What does the – sign mean?
Example 1: Interpretation of Results

I₁ = 2.0A, I₂ = -3.0A, I₃ = -1.0A,

Actual situation
Note how the positive current Typically is pushed away from positive terminal of the emf
Example 1 Again: Different Initial Directions

Different initial direction for $I_1$, $I_2$

- Apply Kirchhoff’s Rules:
  
  Junction c:
  
  $0 = I_3 + I_1 + I_2$

  Loop abcda:
  
  $\varepsilon_1 + I_1R_1 - I_3R_3 = 0$

  Loop befcb:
  
  $-\varepsilon_2 - I_1R_1 - \varepsilon_1 + I_2R_2 = 0$

  Solving three equations:
  
  $I_1 = -2.0A$, $I_2 = +3.0A$, $I_3 = -1.0A$

  Same effective result as in previous slide