Gravity

Newton: "If apple falls toward Earth, why not Moon?"

apple: \( F = mg \)
\[ a = \frac{F}{m}, \text{ but } g \text{ may not be same at moon, may depend on distance } r \]

moon: \( a = \frac{v^2}{r} = \omega^2 \cdot r \) \( \omega = \frac{2\pi}{\text{month}} \)

\[ v = \frac{2\pi}{\text{day}} \cdot \frac{27.3 \text{ days} + 2 \text{hr} \cdot 3600 \text{ sec}}{24 \text{ hr}} = 3.8 \times 10^8 \text{ m} \]

\[ a = 2.7 \times 10^{-3} \text{ m/s}^2 \]

\( g \) (6.4 \times 10^6 \text{ m}) = 9.8 \text{ m/s}^2

\( g \) (2.8 \times 10^8 \text{ m}) = 2.7 \times 10^{-3} \text{ m/s}^2

ratio
\[ \frac{g(6.4 \times 10^6 \text{ m})}{g(2.8 \times 10^8 \text{ m})} = \frac{9.8 \text{ m/s}^2}{2.7 \times 10^{-3} \text{ m/s}^2} = 3.6 \times 10^10 \]

what if \( g \propto \frac{1}{r^2} \)? \( \left( \frac{3.8 \times 10^8}{6.4 \times 10^6} \right) = 59 \neq 3,600 \) ?

NO

what if \( g \propto \frac{1}{r^2} \)? \( \left( \frac{3.8 \times 10^8}{6.4 \times 10^6} \right)^2 = 3,500 \approx 3,600 \)

\[ F = \frac{m \cdot 9.8 \text{ m/s}^2}{r^2} \left( \frac{6.4 \times 10^6 \text{ m}}{r} \right)^2 \]
we're not done! Since force is equal/opposite pair, mass dependence has to be both objects

\[ F = m \frac{9.8 \text{ m/s}^2}{M} \left( \frac{6.4 \times 10^6 \text{ m}}{r} \right) \]

Newton did not know mass of Earth, and you can't know it just from orbit data

\[ F = G \frac{M m}{r^2} \]

* Explains elliptical orbits!

\[
G = \frac{9.8 \text{ m/s}^2}{(6.4 \times 10^6 \text{ m})^2} \text{ Universal Gravitation Constant} \\
= 6.7 \times 10^{-11} \text{ N m}^2/\text{kg}^2
\]

If you can separately measure G

\[ \rightarrow \text{ you measure mass of Earth!} \]

Cavendish (1798) "Experiments to determine the density of the Earth"

got G for within 1%!