8.11. \[ \frac{n_2}{n_1} = \frac{g_2}{g_1} \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = \frac{g_2}{g_1} \frac{e^{-(E_2 - E_1)/kT}}{n_2/n_1} = \frac{(E_2 - E_1)/kT}{k \ln \left( \frac{g_2}{g_1} \right)} \ln \left( \frac{n_1}{n_2} \right) \]

\[ T = \frac{E_2 - E_1}{k \ln \left( \frac{g_2}{g_1} \right) \ln \left( \frac{n_1}{n_2} \right)} \left( \frac{8.617 \times 10^{-5} \text{eV/K}}{10.2 \text{eV}} \right) = 7790 \text{K} \]

8.12. \[ \frac{n_2}{n_1} \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT} = \frac{3}{1} e^{- \left( \frac{4 \times 10^{-3} \text{eV}}{(8.617 \times 10^{-5} \text{eV/K})(300 \text{K})} \right)} = 0.155 \]

8-34. Approximating the nuclear potential with an infinite square well and ignoring the Coulomb repulsion of the protons, the energy levels for both protons and neutrons are given by \( E_n = \frac{n^2 \hbar^2}{8mL^2} \) and six levels will be occupied in \(^{22}\text{Ne}\), five levels with 10 protons and six levels with 12 neutrons.

\[ E_F (\text{protons}) = \frac{(5)^2(1240 \text{MeVfm})^2}{8(1.0078u \times 931.5 \text{MeV/u})(3.15 \text{fm})^2} = 516 \text{MeV} \]

\[ E_F (\text{neutrons}) = \frac{(6)^2(1240 \text{MeVfm})^2}{8(1.0087u \times 931.5 \text{MeV/u})(3.15 \text{fm})^2} = 742 \text{MeV} \]

\( \langle E \rangle (\text{protons}) = (3/5)E_F = 310 \text{MeV} \)

\( \langle E \rangle (\text{neutrons}) = (3/5)E_F = 445 \text{MeV} \)

As we will discover in Chapter 11, these estimates are nearly an order of magnitude too large. The number of particles is not a large sample.

8-35. \[ \frac{N_o}{N} = 1 - \left( \frac{T}{T_o} \right)^{3/2} \quad \text{(Equation 8-76)} \]

(a) \[ \frac{N_o}{N} = 1 - \left( \frac{T_o/2}{T_o} \right)^{3/2} = 1 - \left( \frac{1}{2} \right)^{3/2} = 0.646 \]

(b) \[ \frac{N_o}{N} = 1 - \left( \frac{T_o/4}{T_o} \right)^{3/2} = 1 - \left( \frac{1}{4} \right)^{3/2} = 0.875 \]
For a one-dimensional well approximation, $E_n = \left(\frac{n^2 \hbar^2}{8mL^2}\right)$. At the Fermi level $E_F$, $n = N/2$, where $N$ = number of electrons.

$$E_F = \frac{(N/2)^2 \hbar^2}{8mL^2} = \frac{\hbar^2}{32m} \left(\frac{N}{L}\right)^2$$

where $N/L$ = number of electrons/unit length,

i.e., the density of electrons. Assuming 1 free electron/Au atom,

$$\frac{N}{L} = \left[\frac{N_d \rho}{M}\right]^{1/3} = \left[\frac{6.02 \times 10^{23} \text{ electrons/mol} \left(19.32 \text{ g/cm}^3 \times 10^2 \text{ cm/m}^3\right)^3}{197 \text{ g/mol}}\right]^{1/3} = 3.81 \times 10^9 \text{ m}^{-1}$$

$$E_F = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2 (3.81 \times 10^9 \text{ m}^{-1})^2}{8\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)} = 4.7 \text{ eV}$$

This is the energy of an electron in the Fermi level above the bottom of the well. Adding the work function to such an electron just removes it from the metal, so the well is $5.47 \text{ eV} + 4.8 \text{ eV} = 10.3 \text{ eV}$ deep.

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(a) $N = \sum_i n_i f_0(E_0) f_1(E_1)$ (with $g_0 = g_1 = 1$)

$= C e^{0} C e^{-e/kT} = C(1 + e^{-e/kT})$

So, $C = \frac{N}{1 + e^{-e/kT}}$

(b) $\langle E \rangle = \frac{0 \cdot n_0 + e n_1}{N} = \frac{\epsilon C e^{-e/kT}}{N} \cdot \frac{N e^{-e/kT}}{(1 + e^{-e/kT})N} = \frac{\epsilon e^{-e/kT}}{1 + e^{-e/kT}}$

As $T \to 0$, $e^{-e/kT} = 1/e^{-e/kT} \to 0$, so $\langle E \rangle \to 0$

As $T \to \infty$, $e^{-e/kT} = 1/e^{-e/kT} \to 1$, so $\langle E \rangle \to \epsilon/2$

(c) $C_V = \frac{dE}{dT} = \frac{d\langle N \langle E \rangle \rangle}{d\langle E \rangle} = \frac{d}{dT} \left(\frac{N e^{-e/kT}}{1 + e^{-e/kT}}\right)$

$= \frac{N e^{-e/kT}}{kT^2} \left[\frac{e^{-e/kT}}{(1 + e^{-e/kT})^2} \frac{e^{-e/kT}}{(1 + e^{-e/kT})}\right]$

$= N k \left(\frac{e}{kT}\right)^2 \frac{e^{-e/kT}}{(1 + e^{-e/kT})^2}$
3)(a) In general the total # of particles is \( N = \int_0^\infty n(E) dE \). We have \( g(E) = C E^{1/2} \) and at \( T=0 \),

\[
F_{FD} = \frac{1}{e^{(E-E_F)/kT}+1} = \begin{cases} 
1 & E < E_F \\
0 & E > E_F 
\end{cases}
\]

so

\[
n(E) = \begin{cases} 
CE^{1/2} & E < E_F \\
0 & E > E_F 
\end{cases}
\]

\[
N = \int_0^\infty n(E) dE = \int_0^{E_F} g(E) dE = \int_{E_F}^{\infty} CE^{1/2} dE = \frac{2}{3} CE_F^{3/2}
\]

so

\[
C = \frac{3}{2} \frac{N}{E_F^{3/2}}
\]

(b) Now calculate the number of particles with \( E > E_F \)

\[
N(E > E_F) = \int_{E_F}^{\infty} n(E) dE = \int_{E_F}^{\infty} \frac{CE^{1/2}}{e^{(E-E_F)/kT}+1} dE
\]

Make the approximation (in the integral) that \( E^{1/2} \approx E_F^{1/2} \). This is justified by the fact that \( F_{FD} \) falls rapidly to zero as \( E \) increases above \( E_F \). Then

\[
N(E > E_F) \approx \left( \frac{3}{2} \frac{N}{E_F^{3/2}} \right) E_F^{1/2} \int_{E_F}^{\infty} \frac{1}{e^{(E-E_F)/kT}+1} dE
\]

Let \( x = (E-E_F)/kT \Rightarrow dx = dE/kT \)

The lower integration limit is \( E = E_F \), giving \( x = 0 \) so

\[
N(E > E_F) \approx \frac{3}{2} \frac{N}{E_F} kT \int_0^\infty \frac{dx}{e^x+1} = \frac{3}{2} \frac{N}{E_F} kT \ln 2
\]

So

\[
\frac{N(E > E_F)}{N} = \frac{3}{2} \ln 2 \left( \frac{kT}{E_F} \right)
\]

Here \( kT \approx 0.025 \text{ eV} \) and \( E_F \) is typically a few eV so only a small fraction of the electrons have \( E > E_F \).
6) In the cavity \( u(f) = \frac{8\pi}{c^3} \frac{hf^3}{e^{hf/kt}-1} \). The spontaneous emission rate (per atom) is \( A = \frac{8\pi}{c^3} hf^3 B \) and the stimulated emission rate is \( B u(f) \). \( \therefore \) we want

\[
B \frac{8\pi}{c^3} \frac{hf^3}{e^{hf/kt}-1} = A = \frac{8\pi}{c^3} hf^3 B
\]

\[
\Rightarrow \frac{1}{e^{hf/kt}-1} = 1 \quad \Rightarrow \quad e^{hf/kt} - 1 = 1
\]

\[
\frac{hf}{kt} = \ln 2 \quad \boxed{T = \frac{hf}{k \ln 2}}
\]

For ordinary visible-light photons this would be a very high temperature. For example \( hf = 2eV \) \( \Rightarrow \) \( 3.35 \times 10^4 K \).