Problems to be handed in:

1) The wave function of a particle in the ground state of a harmonic oscillator potential is given by

\[ \psi_0(x) = A_0 e^{-\alpha x^2} \]

where \( \alpha = \sqrt{\text{km}/2\hbar} \).

   (a) Find the normalization constant and then compute \( \langle x^2 \rangle \).
   (b) What do you get for the expectation value of the potential energy?

2) Using the definition of the momentum operator, \( \hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x} \) calculate \( \langle p \rangle \) and \( \langle p^2 \rangle \). What do you get for the expectation value of the kinetic energy? Does your result make sense given the potential energy that you found in problem 1?

3) The wave function

\[ \Psi(x,t) = \left[ \frac{\alpha}{2\pi} \right]^\frac{1}{4} \left[ e^{-i\omega t/2} + 2\sqrt{\alpha} xe^{-3i\omega t/2} \right] e^{-\alpha x^2} \]

is a normalized, time dependent solution to the full Schrodinger wave equation constructed from the eigenfunctions \( \psi_0(x) \) and \( \psi_1(x) \). Find \( \langle x \rangle \) as a function of time for this wave function. You should be able to write your result in the form \( \langle x \rangle = A \cos \omega t \) where \( A \) is a constant that represents the amplitude of the motion of the wave packet.

4) A particle of mass \( m \) and energy \( E \) traveling in the positive \( x \) direction encounters a potential step of height \( V_0 \), where \( V_0 > E \). Assuming that \( V(x) = 0 \) for \( x < 0 \) and \( V(x) = V_0 \) for \( x > 0 \) write down the form of the wave functions for \( x < 0 \) and \( x > 0 \). Match the two solutions at \( x = 0 \) and show that there is no energy quantization. Find the reflection coefficient \( R \).

5) Using the results given in the text (Eqs. (6-68) and (6-69)) for the step potential with \( V_0 < E \), calculate the reflection coefficient for

- \( E = 1.2 V_0 \)
- \( E = 2.0 V_0 \)
- \( E = 10 V_0 \)

6) Using formulas given in class, find the transmission probability for particles incident on a rectangular potential barrier under the following conditions:

   (a) electrons with \( E = 3 \text{ eV} \) on a barrier 3.2 eV high and 1.0 nm wide;
   (b) electrons with \( E = 3 \text{ eV} \) on a barrier 5.0 eV high and 1.0 nm wide;
   (c) protons with \( E = 3 \text{ eV} \) on a barrier 5.0 eV high and 1.0 nm wide.

7) TL problem 6-59.