Review for the third test

Goals:

- Review Chapters 14-17

- Third test on Thursday at 7:15 pm.

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Chapter 14, SHM

The general solution is: \( x(t) = A \cos (\omega t + \phi) \)

with \( \omega = (k/m)^{1/2} \) and \( \omega = 2\pi f = 2\pi /T \)
Energy of the Spring-Mass System

Potential energy of the spring is
\[ U = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \cos^2(\omega t + \phi) \]

The Kinetic energy is
\[ K = \frac{1}{2} m v^2 = \frac{1}{2} k A^2 \sin^2(\omega t + \phi) \]

Damped oscillations

For small drag (under-damped) one gets:
\[ x(t) = A \exp(-bt/2m) \cos(\omega t + \phi) \]
Exercise

What are the amplitude, frequency, and phase of the oscillation?

\[ x(t) = A \cos (\omega t + \phi) \]

\[ A = 0.2 \text{ m} \quad T = 2 \text{ sec} \quad f = 1/T = 0.5 \text{ Hz} \]

\[ \omega = 2\pi f = \pi \text{ rad/s} \]

\[ \phi = \pi \text{ rad} \]

Chapter 15, fluids

\[ F = P_0 A + Mg \]

\[ P = P_0 + \rho g y \]
**Pressure vs. Depth**

- In a connected liquid, the pressure is the same at all points through a horizontal line.

![Diagram of pressure vs. depth]

**Buoyancy**

\[ F_2 = P_2 \text{ Area} \]
\[ F_1 = P_1 \text{ Area} \]
\[ F_2 - F_1 = (P_2 - P_1) \text{ Area} \]
\[ \approx g(y_2 - y_1) \text{ Area} \]
\[ \approx g \rho V_{\text{object}} \]
\[ \approx \text{weight of the fluid displaced by the object} \]
**Pascal’s Principle**

Any change in the pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel.

**Hydraulics, a force amplifier**

\[ \frac{P_1}{A_1} = \frac{P_2}{A_2} \]

\[ \frac{F_1}{A_1} = \frac{F_2}{A_2} \]

\[ \frac{A_2}{A_1} = \frac{F_2}{F_1} \]

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**Exercise**

\[ m_1 = 10 \text{ kg} \]
\[ r_1 = 0.1 \text{ m} \]

\[ m_2 = ? \text{ kg} \]
\[ r_2 = 1 \text{ m} \]

\[ P_1 = P_2 \]

\[ \frac{m_1 g}{A_1} = \frac{m_2 g}{A_2} \]

\[ m_2 = m_1 \frac{A_2}{A_1} = 1000 \text{ kg} \]
What is the pressure at the bottom of the container?

\[ P_A = 1 \text{ atm} + \frac{m_1 g}{A_1} + \rho gh \]

Continuity equation

\[ A_1 v_1 : \text{units of } m^2 \text{ m/s} = \text{volume/s} \]
\[ A_2 v_2 : \text{units of } m^2 \text{ m/s} = \text{volume/s} \]

\[ A_1 v_1 = A_2 v_2 \]
Energy conservation: Bernoulli’s eqn

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]

\[ P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant} \]

Chapter 16, Macroscopic description

- Ideal gas law

\[ P V = n R T \]
\[ P V = N k_B T \]

\( n \): # of moles
\( N \): # of particles
**PV diagrams: Important processes**

- Isochoric process: \( V = \text{const} \) (aka isovolumetric)
- Isobaric process: \( p = \text{const} \)
- Isothermal process: \( T = \text{const} \)

**Work done on a gas**

\[ W = - \text{the area under the } P-V \text{ curve} \]

\[ W = - \int_{\text{initial}}^{\text{final}} P \, dV \]

\[ W = - \int \frac{V_f nRT}{V_i} \, dV \]

\[ W = - nRT \int_{V_i}^{V_f} \frac{1}{V} \, dV \]

\[ W = - nRT \ln \left( \frac{V_f}{V_i} \right) \]
Chapter 17, first law of Thermodynamics

\[ \Delta U + \Delta K + \Delta E_{\text{thermal}} = \Delta E_{\text{system}} = W_{\text{external}} + Q \]

- For systems where there is no change in mechanical energy:

\[ \Delta E_{\text{thermal}} = W_{\text{external}} + Q \]

Exercise

- What is the final temperature of the gas?
- How much work is done on the gas?
Thermal Properties of Matter

Heat of transformation, specific heat

- Latent heat of transformation $L$ is the energy required for 1 kg of substance to undergo a phase change. ($\text{J/kg}$)
  
  \[ Q = \pm ML \]

- Specific heat $c$ of a substance is the energy required to raise the temperature of 1 kg by 1 K. (Units: $\text{J/K kg}$)
  
  \[ Q = M c \Delta T \]
Specific heat for gases

- For gases we typically use molar specific heat (Units: J / K mol)
  \[ Q = n \, C \Delta T \]

- For gases there is an additional complication. Since we can also change the temperature by doing work, the specific heat depends on the path.
  \[ Q = n \, C_v \Delta T \text{ (temperature change at constant V)} \]
  \[ Q = n \, C_P \Delta T \text{ (temperature change at constant P)} \]

\[ C_P = C_v + R \]

Exercise

- How much heat energy is transferred to or from the gas?