Lecture 11

Goals:

- Employ Newton’s Laws in 2D problems with circular motion
- Relate Forces with acceleration

Assignment: HW5, (Chapter 7, 8 and 9 due 10/19)

For Wednesday: Reading through 1st four sections in Ch. 9

Home Exercise
Friction and Motion, Replay

A box of mass $m_1 = 1 \text{ kg}$, initially at rest, is now pulled by a horizontal string having tension $T = 10 \text{ N}$. This box (1) is on top of a second box of mass $m_2 = 2 \text{ kg}$. The static and kinetic coefficients of friction between the 2 boxes are $\mu_s = 1.5$ and $\mu_k = 0.5$. The second box can slide freely (frictionless) on an smooth surface.

Question:

Compare the acceleration of box 1 to the acceleration of box 2?
Home Exercise
Friction and Motion, Replay in the static case

• A box of mass \( m_1 = 1 \text{ kg} \), initially at rest, is now pulled by a horizontal string having tension \( T = 10 \text{ N} \). This box (1) is on top of a second box of mass \( m_2 = 2 \text{ kg} \). The static and kinetic coefficients of friction between the 2 boxes are \( \mu_s = 1.5 \) and \( \mu_k = 0.5 \). The second box can slide freely on a smooth surface (frictionless).

In the case of “no slippage” what is the maximum frictional force between boxes 1 & 2?

\[
N = \mu_s m_1 g = 1.5 \times 1 \text{ kg} \times 10 \text{ m/s}^2 \\
= 15 \text{ N (so \( m_2 \) can’t break free)}
\]

\( f_s = 10 \text{ N} \) and the acceleration of box 1 is

Acceleration of box 2 equals that of box 1, with \( \left| a_2 \right| = \left| T \right| / \left( m_1 + m_2 \right) \) and the frictional force \( f \) is \( m_2 a \)

(Notice that if \( T \) were in excess of 15 N then it would break free)
Exercise Tension example

Compare the strings below in settings (a) and (b) and their tensions.

(a) \[ T_a = \frac{1}{2} T_b \]

(b) \[ T_a = 2 T_b \]

A. \( T_a = \frac{1}{2} T_b \)

B. \( T_a = 2 T_b \)

C. \( T_a = T_b \)

D. Correct answer is not given

Chapter 8
Reprisal of: Uniform Circular Motion

For an object moving along a curved trajectory with constant speed
\( a = a_r \) (radial only)

\[ |a_r| = \frac{v_t^2}{r} \]
Non-uniform Circular Motion

For an object moving along a curved trajectory, with non-uniform speed:
\[ \mathbf{a} = \mathbf{a}_r + \mathbf{a}_T \] (radial and tangential)

For radial acceleration:
\[ |\mathbf{a}_r| = \frac{v_T^2}{r} \]

For tangential acceleration:
\[ |\mathbf{a}_T| = \frac{d|\mathbf{v}|}{dt} \]

Uniform or non-uniform circular motion:

- implies \( |\mathbf{a}_{radial}| = v_{Tang}^2 \cdot \frac{1}{r} \)
- and if there is acceleration there MUST be a net force.
Key steps

- Identify forces (i.e., a FBD)
- Identify axis of rotation
- Apply conditions (position, velocity & acceleration)

Example

Consider a person on a swing:

When is the magnitude of the tension equal to the weight of the person + swing?

(A) At the top of the swing (turnaround point)

(B) Somewhere in the middle

(C) At the bottom of the swing

(D) Never, it is always greater than the weight

(E) Never, it is always less than the weight
Example

At bottom of swing: $v_T$ is max

$$\frac{F_r}{r} = ma = m v_T^2 / r = T - mg$$

$$T = mg + m v_T^2 / r$$

At top of swing: $v_T = 0$

$$\frac{F_r}{r} = 0$$

$$T = mg \cos \theta$$

$T < mg$

$T > mg$

To find the exact angle using only forces requires explicit knowledge of $\theta$ and a good deal of calculus

Conical Pendulum (Not a simple pendulum)

- Swinging a ball on a string of length $L$ around your head

$(r = L \sin \theta)$

$$\sum F_r = ma_r = T \sin \theta$$

$$\sum F_z = 0 = T \cos \theta - mg$$

so

$$T = mg / \cos \theta (> mg)$$

$$ma_r = (mg / \cos \theta) (\sin \theta)$$

$$a_r = g \tan \theta = v_T^2 / r$$

$$\rightarrow v_T = (gr \tan \theta)^{1/2}$$

Period:

$$t = 2\pi r / v_T = 2\pi (r \cot \theta / g)^{1/2}$$

$$= 2\pi (L \cos \theta / g)^{1/2}$$
Conical Pendulum (very different)

- Swinging a ball on a string of length L around your head axis of rotation

Period:
\[ t = \frac{2\pi r}{v_T} = 2\pi \left( \frac{r \cot \theta}{g} \right)^{1/2} \]
\[ = 2\pi \left( \frac{L \cos \theta}{g} \right)^{1/2} \]
\[ = 2\pi \left( \frac{5 \cos 5}{9.8} \right)^{1/2} = 4.38 \text{ s} \]
\[ = 2\pi \left( \frac{5 \cos 10}{9.8} \right)^{1/2} = 4.36 \text{ s} \]
\[ = 2\pi \left( \frac{5 \cos 15}{9.8} \right)^{1/2} = 4.32 \text{ s} \]

Another example of circular motion
Loop-the-loop 1

A match box car is going to do a loop-the-loop of radius r.
What must be its minimum speed \( v_t \) at the top so that it can manage the loop successfully?
Orbiting satellites \( v_T = (gr)^{1/2} \)

Net Force:
\[
ma = mg = mv_T^2 / r
\]
\[
gr = v_T^2
\]
\[
v_T = (gr)^{1/2}
\]

The only difference is that \( g \) is less because you are further from the Earth’s center!

Geostationary orbit
Geostationary orbit

- The radius of the Earth is ~6000 km but at 36000 km you are ~42000 km from the center of the earth.

- \( F_{\text{gravity}} \) is proportional to \( r^{-2} \) and so little \( g \) is now \( \sim 10 \text{ m/s}^2 / 50 \)
  - \( v_T = (0.20 \times 42000000)^{1/2} \text{ m/s} = 3000 \text{ m/s} \)
- At 3000 m/s, period \( T = 2\pi r / v_T = 2\pi 42000000 / 3000 \text{ sec} = 90000 \text{ sec} = 90000 \text{ s/3600 s/hr} = 24 \text{ hrs} \)

- Orbit affected by the moon and also the Earth’s mass is inhomogeneous (not perfectly geostationary)

- Great for communication satellites
  (1st pointed out by Arthur C. Clarke)

Loop-the-loop 1

To navigate the top of the circle its tangential velocity \( v_T \) must be such that its centripetal acceleration at least equals the force due to gravity. At this point N, the normal force, goes to zero (just touching).

\[
F_r = ma = mg = mv_T^2/r
\]

\[
v_T = (gr)^{1/2}
\]
Loop-the-loop 2

The match box car is going to do a loop-the-loop. If the speed at the bottom is \( v_B \), what is the normal force, \( N \), at that point?

Hint: The car is constrained to the track.

\[
F_r = ma = mv_B^2/r = N - mg
\]

\[
N = mv_B^2/r + mg
\]

Loop-the-loop 3

Once again the car is going to execute a loop-the-loop. What must be its minimum speed at the bottom so that it can make the loop successfully?

This is a difficult problem to solve using just forces. We will skip it now and revisit it using energy considerations later on…
Home Exercise

Swinging around a ball on a rope in a “nearly” horizontal circle over your head. Eventually the rope breaks. If the rope breaks at 64 N, the ball’s mass is 0.10 kg and the rope is 0.10 m. How fast is the ball going when the rope breaks? (neglect mg contribution, 1 N << 40 N)

\[ F_r = m \frac{v_T^2}{r} \approx T \]

\[ v_T = (r \frac{F_r}{m})^{1/2} \]

\[ v_T = \sqrt{\frac{0.10 \times 64}{0.10}} \text{ m/s} \]

\[ v_T = 8 \text{ m/s} \]

UCM: Acceleration, Force, Velocity vectors

Physics 207 – Lecture 11
Recap

Assignment: HW5,

For Wednesday: Finish reading through 1st four sections in Chapter 9