This is more tutorial-like than a problem and leads you through a conceptual development of Bernoulli’s equation using the ideas of Newton’s 2nd law and energy. You are going to draw free-body diagrams, calculate Work, and consider kinetic and potential energy. The learning goals for this exercise are to see Bernoulli’s equation as a statement of energy conservation and remove some of the mystery of the pressure drop in a venturi.

**Part I: The Venturi**

Consider a non-viscous and incompressible fluid flowing through a horizontal pipe that changes diameter so that the cross-sectional area is reduced to \( \frac{1}{4} \) its original value as shown. The fluid flows from left to right and small cubic elements of fluid are drawn at three locations.

1) If the velocity at point A \( (v_A) \) is \( v \), then what is the velocity at C \( (v_C) \)? Explain your reasoning.

   ![Free-body diagram of fluid element at each lettered location](image)

   a) What is the velocity at point B \( (v_B) \) exactly halfway through the constriction where the tube diameter \( \frac{3}{4} \) the diameter at point A? Explain your reasoning. Hint: Area is proportional to the diameter squared.

2) Draw a free-body diagram of the fluid element at each lettered location. Include the direction of the acceleration and the net force if they are non-zero. You may omit forces not in the left or right direction. (All six faces of the cube have pressure on them.)

   ![Free-body diagrams](image)

   a) Are the forces pointing left and right the same on each free-body diagram? Explain briefly.
3) What gives rise to the forces responsible for the acceleration of the fluid? Explain your reasoning.

4) On which side of the cube is the pressure higher during the acceleration? Explain your reasoning.

5) Do you agree or not agree with the following statement.

“The forces mentioned in 3 do Work on the fluid element and that Work is equal to the element’s change in kinetic energy.”

Explain your reasoning and consider how you might compute this Work from the change in kinetic energy of the fluid element and relate that to the change in pressure.

6) How does the pressure at point A compare to that at point C? Hint: If the kinetic energy increases something had to provide it. That thing must have less energy now.
Part II: Potential Energy, Pressure, and KE

Consider a non-viscous and incompressible fluid flowing out a horizontal pipe at the bottom of a large tank filled to height $h$ as shown. Small cubic elements of fluid are drawn at three locations. The two on the left represent a water balloon dropped from the same height.

1) Compare the potential energies of the fluid elements at the various locations. Where are they the same, different, larger, or smaller?

2) Compare the pressures on the fluid elements at the various locations. Where are they the same, different, larger, or smaller? Explain your reasoning. If you’re stuck about how to handle air pressure, ask your TA.

The tank is very large so an element’s velocity at A and B are pretty much zero as is the velocity of the water balloon at 1 that is dropped from rest.

3) Use the ideas in Part I and the change in PE to compare the velocities at 2 and C. Explain your reasoning.
2) A large quantity of water (density = 1.00 gm/cm³ or 10³ kg/m³) is poured into an unusual setup in which there is a loudspeaker mounted on the left hand side of a horizontal pipe attached to a large reservoir. The diameter of the pipes and other dimension are given in the figure. For parts (a) and (b) the water is assumed to behave as a Newton fluid.

(a) With the speaker off, what is the pressure (in Pascals) in the horizontal tube?

(b) Now the power to the speaker is switched on and the x motion of the speaker is given by \( x(t) = A \cos(\omega t) \) where the amplitude, \( A = 0.020 \text{ m} \), and the frequency, \( f \), is set to 0.50 Hertz. The water moves in sync with the speaker. The height of the water in the vertical column does not visibly change. What is the maximum pressure difference, \( p = p_1 - p_2 \), between the pressure in the 2.0 m diameter section & and 0.50 m section?
3) One liter of water (1000 cm$^3$ with $\rho = 1.00$ gm/cm$^3$ or $10^3$ kg/m$^3$) is poured into a uniform U-shape pipe with a $1.00 \times 10^{-4}$ m$^2$ square cross section bore and a shape as shown in the figure at right. The drawing is not to scale.

(a) When the water is at equilibrium, what is the pressure (in Pascals) at the bottom of the tube? You should ignore the details of the bend and only consider the straight sections of the tube in your calculation.

(b) Now you gently blow into one side of the tube and the water surface, after coming to equilibrium, sinks down 0.010 m from its starting height. What relative pressure (i.e., change in pressure), in Pascals, did you apply?

(c) You now double the pressure of part (b) and find that the fluid sinks down twice as far. After this you release the pressure and the fluid sloshes back and forth. Assuming that water behaves as a perfect Newtonian fluid, at what angular frequency does it oscillate?