Simple Harmonic Motion

Group Problem 8  Name_____________________

A) A horizontal spring-mass system of spring constant $k$ and mass $m$ oscillates on a frictionless surface but it is immersed in a fluid that produces a drag force proportional to the velocity. We have seen what the motion is without drag and here we ask you to build upon that to answer the following questions about the effect of friction on the motion.

1) How does drag affect the period and frequency?

2) How does drag affect the amplitude over time?

To answer these questions we need to find $x(t)$, the solution to Newton’s 2nd law that describes position as a function of time. You will work as a group to accomplish this task as outlined below. To make life easier for you and the TAs work under the following assumptions and initial conditions.

a) Assume the spring obeys Hooke’s law. Also, to better visualize the form of the solution, assume the spring force is generally much larger than the drag force so that there are many oscillations.

b) Assume the drag force can be modeled as a constant times the velocity, $f_{drag} = -bv$ where the minus sign means that $f$ and $v$ point in opposite directions.

c) Start with the initial condition shown in the figure which can be taken starting from rest or starting at a turnaround point; and consider only the motion to the next turnaround point.

Follow the steps below and the group instructions. It is more important to experience the problem solving exercise than to arrive at the solution. Resist the urge to look in your text!

1) As a group discuss and predict what an $x$ vs. $t$ plot would look like for damped harmonic motion. This step can help you visualize the mathematical form of the solution. Draw your prediction at right.

2) As a group come to a consensus as to the form of the differential equation that comes from Newton’s second law. That is, $a = \frac{d^2x}{dt^2} = \frac{F_{net}}{m}$ and the job is really to figure out $F_{net}$. Before continuing ask your TA to check your reasoning and your differential equation. To make checking your work easier write your differential equation in the form:

$$\frac{d^2x}{dt^2} = Stuff$$

where $Stuff$ contains $x$, $\frac{dx}{dt}$ and coefficients.
Each group member should guess at the solution to the differential equation, write it down, and differentiate it twice below.

3) Discuss as a group your answers to part 3) and if necessary make another guess at the solution. Hint: Since $\frac{d^2x}{dt^2}$ must yield two terms, a product of two functions might be a useful guess since the product rule for differentiation will give two terms. Write down your solution below.

4) Answer the questions posed on the first page restated below.

a) How does friction affect the period and frequency? Hint: Find $\omega$

   i) Is $\omega$ still a constant or does it vary with time?

   ii) How does $\omega$ depend on $b$?

b) How does friction affect the amplitude over time?
B) A small scale, of mass 1.0 kg, is mounted on top of a massless vertically mounted Hooke’s Law spring with spring constant 200 N/m. A 1.0 kg mass is placed on the scale and now the assembly moves to the new equilibrium position.

a) If the spring is now displaced downwards 0.200 m further and then released from rest, what is the initial frequency of oscillation?

b) What is the initial acceleration of the scale/mass assembly?

c) What does the scale read after T/4 where T is the period?

d) Does the mass always remain in contact with the spring? If yes, then then how long from release before the scale reads a minimum in the weight? If no, then how long after the release [i.e., from part (a)] does the 1.0 kg mass remain in contact with the scale?
C) It has recently become possible to "weigh" DNA molecules by measuring the influence of their mass on a nano-oscillator. The figure shows a thin rectangular cantilever etched out of silicon (density 2300 kg/m$^3$) with a small gold dot at the end. If pulled down and released, the end of the cantilever vibrates with simple harmonic motion, moving up and down like a diving board after a jump. When bathed with DNA molecules whose ends have been modified to bind with gold, one or more molecules may attach to the gold dot. The addition of their mass causes a very slight-but measurable-decrease in the oscillation frequency. A vibrating cantilever of mass $M$ can be modeled as a block of mass $M/3$ attached to a spring. (The factor of $1/3$ arises from the moment of inertia of a bar pivoted at one end.) Neither the mass nor the spring constant can be determined very accurately—perhaps to only two significant figures—but the oscillation frequency can be measured with very high precision simply by counting the oscillations. In one experiment, the cantilever was initially vibrating at exactly 12 MHz ($1$ MHz = $10^6$ Hz). Attachment of a DNA molecule caused the frequency to decrease by 58 Hz.

1) What was the mass of the DNA?