2) You are a rocket designer testing a new escape system. You launch the 7.0 kg assembly straight up in the air. It flies up 50 m and, at the top of its trajectory, the prototype system explodes into three fragments. Two of the fragments are equal in mass (each of 2.0 kg) and leave explosion point with equal speeds (of 6.0 m/s). Both of these pieces head towards the ground and there is an angle of 120° between their respective velocity vectors. The plane formed by these two velocity vectors is perfectly vertical. In addition, each of the two pieces hits the level ground at the same moment in time (you should ignore the effects of air resistance).

To what height does the third piece rise?

\[
\begin{align*}
| \vec{P}_2 | &= | \vec{P}_1 | = 12 \text{ kg m/s} \\
\vec{P}_y^i &= \vec{P}_y^f \\
V_{y3} &= 0 \quad (\text{AT TOP}) \\
P_{y3}^i &= 0 = P_{y3}^f = m_3 V_{y3} - 2 \cdot 12 \text{ kg m/s} \cos 60° \\
0 &= 3 \text{ kg} V_{y3} - 12 \text{ kg m/s} \\
V_{y3} &= 4 \text{ m/s} \\
m_3 \frac{1}{2} V_{y3}^2 &= 2 \cdot 16 \text{ J} \\
V_{y3} &= \sqrt{\frac{32 \text{ J}}{4.0 \cdot 10 \text{ kg m/s}^2}} = 0.75 \text{ m} \\
\gamma_3^{\text{NET}} &= (50 + 0.75) \text{ m} = 51 \text{ m} 
\end{align*}
\]
3) The only force acting on a 2.0 kg object moving along the x-axis. Notice that the plot is force vs time.

If the velocity \( v_x \) is \(+2.0 \text{ m/s}\) at 0 sec, what is \( v_x \) at 4.0 s?

\[
\Delta P = m \Delta v = \int F \cdot dt
\]

\[
v_f = v_i + \Delta v
\]

\[
\Delta P_{AB} = -8 \text{ kg m/s}
\]

\[
\Delta P_{BC} = -4 \text{ kg m/s}
\]

\[
\Delta P_{CD} = 16 \text{ kg m/s}
\]

\[
\Delta P = 4 \text{ kg m/s}
\]

\[
\Delta v = +2 \text{ m/s}
\]

\[
v = 4.0 \text{ m/s}
\]
4) Three masses, as shown below, are placed on a horizontal frictionless table. Initially the two larger masses are at rest and the 3.0 kg mass is traveling to the right. There is a collision and afterward the 3.0 kg mass is seen to be at rest while the two 12 kg masses move along the indicated paths. Force sensors, in each 12 kg mass, produce identical plots as shown (for forces along the respective velocity vectors).

(A) What is the final speed of a 12 kg mass?

\[
\begin{align*}
\text{PLOT REFLECTS } |\Delta \vec{p}| & \text{ for each 12 kg mass} \\
\int \vec{F}_i \, dt & = \vec{F} \\
|\vec{F}| & = 12 \text{ Ns} \\
|\vec{p}_f - \vec{p}_i| & = 12 \text{ kg m/s} \\
|\vec{v}_f| & = 1.0 \text{ m/s}
\end{align*}
\]

Collision cons. Momentum

\[ P_{i, x} = P_{f, x} \]

(B) How much mechanical energy (if any) was lost in the collision?

\[
\begin{align*}
3 \text{ Kg} \, V_x & = 0 + 12 \text{ Ns} \cos 60^\circ + 12 \text{ Ns} \cos 60^\circ = 12 \text{ Ns} \\
V_{x_i} & = 4.0 \text{ m/s} \\
KE_i & = \frac{1}{2} \times 3.0 \times (4.0)^2 J = 24 J \\
KE_f & = \frac{1}{2} \times 12 \times 1.0^2 J = 12 J
\end{align*}
\]

\[ 12 J \text{ lost} \]
5) (optional, for fun) You are developing a new model rocket that is tethered to a central frictionless pivot via a thin massless wire, 4.0 meters in length. The wire will break if the wire’s tension exceeds 172.0 N. The rocket, of mass 10.0 kg, starts from rest and undergoes horizontal circular motion on surface with a coefficient of sliding friction of 0.10. You can assume \( g = 10 \text{ m/s}^2 \). This rocket design generates a constant 30.0 N force which is always directed \( \text{tangent} \) to the circular motion. The rocket’s mass does not change and there are no other frictional forces.

a) What is the initial acceleration of the rocket?

b) When the rocket is halfway around, the wire then encounters a small rigid pin and, at that moment, the rocket’s motor turns off. What is the maximum distance from the center pivot that the rigid pin can be placed so that wire will not break?

\[
V_{\text{start}} = 0
\]

\[
\vec{F}_i = m \vec{a}_i \quad \Rightarrow \quad F_{\text{y}} = 30 \text{ N} = 10 \text{ kg} \quad \text{a}_y
\]

\[
\text{a}_y = 3.0 \frac{\text{m}}{\text{s}^2}
\]

\[
\vec{a} = 3.0 \frac{\text{m}}{\text{s}^2} \hat{\text{y}}
\]

\[
\begin{align*}
S &= \pi R \left( \frac{1}{2} \text{ circumference} \right) \\
S &= \frac{1}{2} a_y \Delta t^2 \\
\frac{2 \cdot \pi \cdot 4}{S} &= \Delta t^2 \\
\Delta t &= \sqrt{\frac{8 \pi}{3}} \quad 2.89 \text{ s} \\
V_T &= a_y \Delta t = 3 \cdot 2.89 \frac{\text{m}}{\text{s}} = 8.7 \frac{\text{m}}{\text{s}}
\end{align*}
\]

\[
172 \text{ N} \geq T = m \frac{V_T^2}{r}
\]

\[
R_{\text{MIN}} = \sqrt{\frac{m \cdot (8.7)^2}{172}} = \sqrt{4.74} = 2.17 \text{ m}
\]

\[
\delta = R - r_{\text{MIN}} = 2.7 \text{ m}
\]