Today’s Topics

- Capacitance (Ch. 26)
- Energy stored in capacitors
- Combinations of capacitors
- Dielectric and Capacitance

- Previews: Energy density, electric dipole moment, torques, dielectric material.
Lecture 6 Review: Charging A Capacitor

Electric potential energy gained:

\[ U = \int du = \int (-\Delta V)(-dq) = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} \]

After charging the capacitor stores potential energy:

\[ U = \frac{1}{2} \frac{Q^2}{C} \]
Lecture 6 Review: Discharging A Capacitor

- Potential energy released:

\[ U = \int dU = \int \Delta V (-dq) = \int_0^\infty -\frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C} \]

- the originally charged capacitor has potential energy:

\[ U = \frac{1}{2} \frac{Q^2}{C} \]
Energy Stored In A Capacitor

- Show that energy required to charge a capacitor to a charge Q is \( U = \frac{1}{2} C(\Delta V)^2 \)

Solution: See previous two slides → energy stored in a capacitor is \( U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C(\Delta V)^2 \)

- Important: This energy is actually stored in the form of the electric field.

- Energy density of an Electric Field:
  \[ u_E = \frac{U}{\text{volume}} = \frac{U}{Ad} = \frac{1}{2} \frac{Q^2}{(CA)d} \]
  \[ = \frac{1}{2} \frac{Q^2}{(\varepsilon_0 A)^2} \]

  \( u_E = \frac{1}{2} \varepsilon_0 E^2 \)
Quick Quiz

- A parallel plate capacitor is holding a charge $q$. When the plates are pulled apart (i.e. $d$ increases), what happens to the energy stored in it?

1. Increases
2. Decreases
3. Stays the same

Arguments:
- Volume increases while $E$ almost the same
- Work done to it when being pulling apart
Electric Dipole Moments

- Electric dipole moment $\mathbf{p}$.

$$\mathbf{p} = q \mathbf{d}$$

- Electric dipole moment in constant $E$ field

Note: $\mathbf{P}$ is a vector

- Net Force
  $$\sum \mathbf{F} = 0$$

- Net Torque
  $$\mathbf{\tau} = \mathbf{p} \times \mathbf{E}$$

- Potential energy
  $$U = -\mathbf{p} \cdot \mathbf{E}$$
Quick Quiz

Assuming a dipole moment can only rotate in an electric field, which of the following configurations is stable?

1. 45°

2. Points towards the E field

3. Points against the E field

4. Points normal to the E field
Dielectrics In External Field

- Dielectric material (dielectrics): Insulators that contain (microscopically) large amount of dipole moments
  - permanent
  - induced.
- Alignment of permanent dipoles in external field

Note: In an external field $E_0$, the induced field $E_{ind}$ is always opposite to $E_0$
When no dielectrics inserted

\[ E = E_0 \rightarrow \Delta V_0 = E_0 d \]

\[ \Rightarrow \text{capacitance } C_0 = \frac{Q}{\Delta V_0} \]

After insertion of dielectrics

\[ E_{\text{ind}} = -\alpha E_0 \]

\[ \Rightarrow \Delta V = Ed = \frac{1}{\kappa} E_0 d = \frac{1}{\kappa} \Delta V_0 \]

\[ \Rightarrow \text{capacitance } C = \frac{Q}{\Delta V} = \kappa C_0 > C_0 \]

\[ \Rightarrow \text{Larger } C \]

\[ \Rightarrow \text{more } Q \text{ stored per } \Delta V \]

\[ \Rightarrow \text{less } \Delta V \text{ per fixed } Q \]

\[ \Rightarrow \text{More energy stored per } \Delta V \]

(recall: \( U = \frac{1}{2} CV^2 \))
Use of Dielectric Material

- Often, a non-conducting dielectric material is inserted in-between conductor sheets to increase the capacitance.
Demo: Insert Dielectrics In Between Conductor Plates

(a) $C_0$, $Q_0$, $\Delta V_0$

(b) $C$, $Q_0$, $\Delta V$
Extra Topic: Accelerators
(Not to be examed)

Definition

Particle accelerator:
A facility where particles are accelerated to very high energy.

Recall: high kinetic energy $\leftrightarrow$ high speed
Particle Energy And Speed

Unit: eV = (1 electron charge) * (1 volt) = 1.6x10^{-19} Joule

c=3x10^8 m/s

<table>
<thead>
<tr>
<th>Kinematic Energy</th>
<th>eV</th>
<th>KeV (10^3 eV)</th>
<th>MeV (10^6 eV)</th>
<th>GeV (10^9 eV)</th>
<th>TeV (10^{12}eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Speed (m/s)</td>
<td>6x10^5 (0.002c)</td>
<td>2x10^7 (0.06c)</td>
<td>2.8x10^8 (0.94c)</td>
<td>3x10^8 (0.999999c)</td>
<td>3x10^8 (1.0c)</td>
</tr>
</tbody>
</table>

Higher energy = faster

Why Accelerating?

High energy ➔
- “smash” fundamental particles
- produce new heavy (.i.e. massive) particles
- see into tiny microscopic world
- new technology (cancer treatment, synchrotron light sources, etc.)
- ...
Accelerate Particles By A Static Electric Field

\[ \Delta E = e \Delta V \]

Unit: \( eV = (1 \text{ electron charge}) \times (1 \text{ volt}) = 1.6 \times 10^{-19} \text{ Joule} \)

- Boiling Water: 0.03 eV per molecule
- TV CRT: 20 KeV per electron
- Nuclear explosion: 2MeV (2x10^6 eV) per nucleon
- Modern Accelerators: \( 10^{11} - 10^{13} \) eV per particle
  \(~ 10^6 \times \text{Nuclear Power per particle}!\)
Types of Accelerators

- Static field based. (eg. CRT, Van de Graff.)
  - Technical difficulty of using static E. Field:
    - Very high voltage difference is difficult to generate and to maintain.
      - Air “break down”
      - Huge electric force/field (Best Van de Graff: $\sim 10^7$eV.)

- Alternating field based.  (alternating: varying periodically in time)
  - Application of combined electric and magnetic field properties (later this semester).
  - General principle: $E = e \Delta V \times N$  (N: do it many times)
    - Must use alternating fields.
  - Often heard names: Cyclotron, Synchrotron, Linac, …
Large Hadron Collider (LHC)
The Most Powerful Accelerator in the World

LHC accelerates protons \((p,p)\) along the 27 km ring and collides protons at very high energy.
Overall view of the LHC experiments.

LHC: Large, powerful, precise, expensive, ... and useful
LHC Is VERY Powerful

- At LHC, each proton is accelerated to 7 TeV (7x10^{12} eV)
  → At collision, 2x7 = 14 TeV of energy is released.

- Recall: Energies scales
  - Boiling Water: 0.03 eV per molecule
  - TV CRT: 20 KeV (2x10^4 eV) per electron
  - Nuclear explosion: 2MeV (2x10^6 eV) per nucleon
  - LHC : 7x10^{12} eV per proton

- Per particle: LHC > 3 million x nuclear reaction !
Wait, Wait, You Mean
Three MILLION Nuclear Bombs?

- Relax! Not at all.
  At LHC, Only about $10^{14}$ protons are involved, while a small nuclear bomb has $10^{25-26}$ reacting nucleons.

- Total energy is much less at LHC. ($< 10^{-4}$ of an A-bomb)

- Still, per particle-wise, modern accelerators are much more powerful than nuclear reactions in terms of energy density.

- Important message, LHC is powerful and safe!
LHC: Beam Pipe And Tunnel
LHC: Transportation Inside the Tunnel

Note:
The tunnel is 27 km long and 150m underground
LHC: Accelerating Cavity
LHC: Superconducting Magnets
Detectors at LHC(1): ATLAS

Cost: $300M

UW involvement: Prof. Wu, Pan, Mellado
Detectors at LHC(2): CMS

UW involvement: Prof. Carlsmith, Dasu, Herndon, Reeder, Smith