Physics 202, Lecture 26

Today’s Topics

- Wave Nature of Lights: Interference
- Light as Waves
- Double-Slit Interference
- Multi-Slit Interference

Reminder: Light and Optics

- Nature of Lights
  - Lights as rays
  - Lights as EM waves: f, λ, φ, v, A, interference …
  - Lights as group of photons

- Optics: Physics of lights
  - Geometric Optics: Treat light as rays (Ch. 35,36)
    - Ray approximation.
  - Wave Optics: Wave properties becomes important.
    Interferences, diffraction…(Ch. 37,38)

Reminder: Light Waves

- Nature of Lights:
  - Rays (classical), EM waves↔, Photons↔.
- Review: Electromagnetic plane waves
  \[ E = E_{\text{max}} \sin(\omega t - kx + \phi) \], \[ B = B_{\text{max}} \sin(\omega t - kx + \phi) \], \[ E/B = c \]
  - As the E component and B component of an EM wave are 100% correlated, we can use just one of them to represent an EM wave.

Quick Reviews

- Superposition Principle (ch. 18).
  - when two waves, \( y_1(x,t) \) and \( y_2(x,t) \) meet, the resulting wave is the algebraic sum of the two waves: \( y(x,t) = y_1(x,t) + y_2(x,t) \)

- Intensity of an EM wave (ch. 34) \( I = \frac{S}{c} = E_{\text{max}}^2 = B_{\text{max}}^2 \)
Useful Math Formulas

- \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta
- \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta

Small angle approximations:
\sin \theta \sim \theta, \tan \theta \sim \theta, \text{ when } \theta \sim 0

- Long distance approximation: \( d \sim L^* \theta \), when \( L >> d \)

Quiz: If the intensity of each incoming light is \( I \), what is the resulting intensity when (1): constructive, (2): destructive?

Interference of Light Waves

- When two light waves meet at certain location, the resulting effect is determined by the superposition (i.e. sum) of the two individual waves
- E.g. Two light waves with same color and amplitude,
  \( E_1 = E_0 \sin(\omega t - kx + \phi_1) = E_0 \sin(\omega t + \phi_1) \)
  \( E_2 = E_0 \sin(\omega t - kx + \phi_2) = E_0 \sin(\omega t + \phi_2) \)

  \( \Delta \phi = \phi_1 - \phi_2 \)

  \( \phi_1 = -kx + \phi_{10}, \quad \phi_2 = -kx + \phi_{20} \)

  \( E = E_1 + E_2 = 2E_0 \cos(\Delta \phi/2) \sin(\omega t + \phi) \)

  Resulting amplitude: \( E_{\text{max}} = 2E_0 \cos(\Delta \phi/2) \)
  - Constructive interference: \( \Delta \phi = 0, \pi, 2\pi, 4\pi, ... \) \( E_{\text{max}} = (+/-)2E_0 \)
  - Destructive interference: \( \Delta \phi = \pi, 3\pi, 5\pi, ... \) \( E_{\text{max}} = 0 \)

  It all depends on \( \Delta \phi \)!

Quiz: If the intensity of each incoming light is 1, what is the resulting intensity when (1): constructive, (2): destructive?

Constructive and Destructive Interference

Resulting amplitude: \( E_{\text{max}} = 2E_0 \cos(\Delta \phi/2) \)

\[ \begin{align*}
\text{(a)} & \quad \text{Constructive, } \Delta \phi = 0\pi, 2\pi, 4\pi, ... \\
\text{(b)} & \quad \text{Destructive, } \Delta \phi = \pi, 3\pi, 5\pi, ... 
\end{align*} \]

Test of the Wave Nature of Light: Double-Slit Experiment

- Rays or Waves?
- Diffraction & interference

If lights behave as rays
- If lights behave as waves

Young’s Famous Double-Slit Experiment
Thomas Young (1803)

See demo

One More Review (ch. 18)
Path Length And Path Length Difference

- Remember a Phy201 problem like this?

- For two interfering waves coming through different paths the phase difference:
  \[ \phi = k(r_1 - r_2) + \Delta \phi_{\text{at the source}} \]
  (why? see board)

  where \( r_1 \) and \( r_2 \) are path lengths,
  \( \Delta r = (r_1 - r_2) \) is called path length difference

Double-Slit Experiment Explained

- The experiment can be easily explained by interference

Constructive, \( \Delta \phi = 0\pi, 2\pi, 4\pi, \ldots \)
Destructive, \( \Delta \phi = \pi, 3\pi, 5\pi, \ldots \)

Quantitatively

\[
\Delta \phi = k(r_1 - r_2) = kd \sin \theta = \frac{2\pi d}{\lambda} \sin \theta
\]

\[
I = I_0 \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)
\]
Double-Slit Experiment Explained

- **Constructive:** $\Delta \phi = 0\pi, 2\pi, 4\pi, \ldots$, or $2m\pi$, $m=0,1,2\ldots$

  \[ \frac{2\pi d}{\lambda} \sin \theta = 2m\pi \quad \Rightarrow \quad d \sin \theta = m\lambda. \]

  **Bright spots**

- **Destructive:** $\Delta \phi = \pi, 3\pi, 5\pi, \ldots$, or $(2m+1)\pi$, $m=0,1,2\ldots$

  \[ \frac{2\pi d}{\lambda} \sin \theta = 2(m+1)\pi \quad \Rightarrow \quad d \sin \theta = (m + \frac{1}{2})\lambda. \]

  **Dark spots**

Multi-Slit Interference

- # secondary maxima = $N - 2$
- Higher $N$ $\Rightarrow$ more suppression on secondary minima
  (Grating: $N>1000$, highly sensitive to $\lambda$, good for measuring $\lambda$.)

Self reading: Phasor Method (Not to be examined)